ModLab: A New MatLab Toolbox for Modal Parameters Identification

Aniello Daniele Marano*

Abstract—The purpose of this study is to introduce ModLab, a new MatLab toolbox for Experimental Modal Analysis.
ModLab is short for Modal (Analysis) Laboratory. ModLab focuses on three different Modal Parameter Estimation processes to derive the structural behaviour of a multidegree-of-freedom system: Line-fitting, Circle-Fitting, and Least-Squares Complex Exponential (LSCE) methods. The first part of the study concerns some cases of application of ModLab as an Experimental Test Simulator.
The structure taken into consideration for the simulation is an aircraft schematized through a simplified model consisting of rod elements. In order to derive the natural modes of vibration of the dummy airplane and the corresponding natural frequencies, different damping conditions and load cases were assumed. The different simulated case studies have revealed a strong consistency of the solutions: results are physically verifiable in reality.
In order to validate and test ModLab also with real structures, an experimental modal analysis was computed. The modal testing was about a bogie skirt used on a high-speed train to reduce rolling and aero acoustic noise to the exterior. Finally, a comparison between ModLab results and TestLab results has provided percent deviations less 1% in most cases.
Therefore, it is concluded that ModLab can be considered a valid alternative to the most widespread commercial modal analysis software.

Keywords— MatLab toolbox, Modal Parameters Identification, Experimental Modal Analysis, Impact Testing.

I. INTRODUCTION

M odal Analysis is the study of the dynamic behavior of a structure subjected to mechanical vibrations. Being system of elastic components, structures and machines respond to external and internal forces with finite deformations and motion. In structural analysis, MA allows the determination of the nature and extent of vibration response levels and then the properties of a structure, constrained or free, excited by internal or external dynamic forcing. Modal Analysis covers a wide range of objectives: identification and evaluation of vibration phenomena; correction and validation of theoretical and prediction models; model integration with other areas of dynamics such as fatigue and acoustics.
From the historical point of view, the principles of MA were

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established starting around 1940 when they found a first application in the determination of natural frequencies and damping levels of aeronautical structures in order to predict and control the self-excited aerodynamic *flutter* problem, a critical issue for the aerospace industry. Nowadays, *Ground Vibration Testing* (GVT) is still carried out on the prototype of every new airplane model designed. The need to solve self-excited vibration problems was also felt in the machine tool industry, where this problem known as *chatter*.
Furthermore, the Fourier series and the study of spectrum analysis laid a solid foundation for the development of modal analysis.
In 1965, the development of the *Fast Fourier Transform* (FFT) algorithm paved the way for application of experimental techniques in structural dynamics. In subsequent years, research efforts were focused on identifying modal data from measured FRF signals and numerous modal parameter identification methods have been proposed, from Single DOF to Multiple DOF, time domain methods to frequency domain methods, single reference to multiple reference, etc.
Since the 1970s, major advances in transducers, electronics and digital analyzers and the advent of digital computers and low cost electronics (sensors and measuring equipment was commercially available) have consolidated modal test techniques and allowed the development of the modern *Experimental Modal Analysis* (EMA).
Nowadays, MA is widely used in various engineering fields for example mechanical and aerospace engineering. For automotive, aircraft and spacecraft structures, the vibrations are directly related to performance, causing malfunction or by creating disturbance or discomfort, including noise but also fatigue and failures. Other important fields of application of the modal analysis are civil engineering for the analysis of seismic stresses or for monitoring of bridges sustaining traffic and strong winds; biomedical engineering and sound engineering; many products of the sports and music industries, such as tennis rackets and golf clubs and even musical instruments.
Because structural vibration problems represent a design limitation for a very wide range of engineering products, it is important that the vibration levels encountered in service be anticipated and brought under satisfactory control.
Mechanical resonance phenomenon is the most important vibration problem that a structure can encounter under

operating conditions. Resonant vibration is caused by an interaction between the inertial and elastic properties of the materials and it occurs when one or more of the natural modes of vibration of a structure is excited. At the natural frequencies, the modes of the structure act as a mechanical amplifier so a small amount of input force can cause a very large response. In order to predict and control structural vibration problem, the resonances of a structure need to be appropriately identified and quantified. A common way to predict vibration problems is to derive the modal parameters of the structure from the measured data. Each mode is indeed defined by a natural frequency, a modal damping and a mode shape. Modal parameters are global properties of a structure. If either the geometry, the material properties or the boundary conditions of a structure change, its modes will change.

EMA or *Modal Testing*, bases on the parameter identification methods and includes both the data acquisition and its subsequent analysis. Using an excitation mechanism, a shaker or a hammer, the structure is excited with a known force and its response due to the applied force is measured through a sensing mechanism. The mechanical response of the structure can be defined in terms of displacement, velocity or acceleration.

The ratio of output response to input force form the Frequency Response Function (FRF). The FRF is a matrix which terms identify input-output measurements: the generic term h_{ij} gives output response at point *"i"* due to an input force applied at point *"j"*.

The measured data are transformed from the time domain to the frequency domain using a FFT algorithm and the FRF's became complex functions valued numbers, with real and imaginary components or, equivalently, magnitude and phase components, and modal data can be extracted from them.

The *Modal Parameter Estimation* process, which is often referred to as *curvefitting*, is implemented using computer software. The analyst must specify the band over which data is extracted, the number of modes contained in the data and the inclusion of residual compensation terms for the estimation algorithm. Finally, the frequency, damping and mode shapes are so extracted from the measurements.

Some of modal parameters identification techniques can be integrated into a lot of pre- and post-processing commercial software, designed to facilitate the study of structural dynamic analysis, through geometry definition, integration with CAD software, structural modification prediction, large models of FE assemblies management, structural modification prediction, mode animation, etc.

ModLab focuses on the *Line-fitting*, *Circle-Fitting*, and *Least-Squares Complex Exponential* (LSCE) methods [1] - [5] in order to derive the structural behaviour of a multidegree-of-freedom system, in two different approaches: as an MA simulator to verify results deriving from the FEM analysis and as an EMA tool. The implementation of the aforementioned methods has been supported by the availability of some functions offered by the free EasyMod package.

II. GETTING INTO MODLAB

A general overview of ModLab is briefly introduced. ModLab is short for Modal (Analysis) Laboratory. ModLab is a MatLab toolbox for EMA, created with the intent to propose an 'easy-to-use' software to investigate the dynamic behaviour of a given structure. ModLab integrates appropriately some of the tools provided by the EasyMod package, increasing it with new routines.

Of particular interest are the integrated functions that include, for example, the possibility of:

- printing in universal file format of the transfer functions;
- assembling of the FRF matrix with the available FRF's;
- plotting FRF's in various formats;
- start some parametric identification method.

The first step to study the dynamic behaviour of a structure is the realization of the geometric model of the structure under examination. This procedure can be performed by means of a processing software or by accessing to geometry data from all major CAD systems. At the end of this step, a *.bdf* format file containing the information about the geometry will be extracted. Then, from the desire to manage through neutral file the structure geometry and, subsequently also the identified modes, it is necessary to obtain a *.neu* format file from the *.bdf* file. A *.neu* format file is a file that contains geometry data, parameter data, materials data and other design information easily exported and imported into CAD software.

The second step is the acquisition of FRF. This process takes place numerically or experimentally. In the first case we proceed with a modal analysis applying the FEM, while in the second case we proceed with modal testing.

Most of the commercial pre and post-processing software packages for modal analysis can import or export a particular extension file (*.uff, .uf* or *.unv*), called *universal file* (UF), defined as data file under ASCII format containing measurement, analysis, units or geometry information, organized in data blocks called 'datasets'. UF format facilitates data transfer between computer aided design (CAD) and computer aided test (CAT) in order to facilitate computer aided engineering (CAE). So it is necessary to perform an intermediate step: obtain a *.unv* format file from the *.pch* file containing the analysis results.

At this point, the modal parameter estimation with ModLab can be performed. When the modal analysis is successfully completed, details of the solution are saved in the output file, what has *.neu* extension. In this file, in addition to the starting information related to the geometry of the structure, you can find a listing of some or all of the calculated natural frequencies and mode shapes.

Because it is not easy to visualise the predicted mode shapes when they appear as tables of data values, it is necessary to import the output file into a post processing software, in order to make the output results very intuitive and readable, via an on-screen 3D display of modal displacements and curvatures.

III. APPROACH TO MODLAB AS AN EXPERIMENTAL MODAL ANALYSIS SIMULATOR

A real structure has infinite degrees of freedom (DOF), so a discretization process is necessary that limits the DOF to a number *n*. The behaviour of a complex system consisting of *n* dofs can be studied as the sum of n simple 1 DOF systems, each of which is characterized by its own natural frequency, damping and vibration mode. The fundamental hypothesis on which the study of the dynamic behaviour of a structure is based is that the system under studying is a linear and invariant time system. This hypothesis is widely verified for most of the real structures dealt with and allows a simplified mathematical treatment.

In general, the dynamic behaviour of a structure can be evaluated using FEM or equivalently with the Laplace domain representation of the system. Both methods use approximations of the physical parameters of mass, damping and stiffness to describe the system. By using Modal parameters estimation techniques, you can estimate the dynamic behaviour of the structure, without any assumption about physical properties and therefore avoiding any errors due to approximations.

The structure taken into consideration for the simulation is an aircraft schematized through a simplified model consisting of rod elements.

Then, the structure is discretized in a number of nodes and fuselage, right and left wings, horizontal and vertical tails are represented through one-dimensional elements that connect the nodes. The nodes form a grid of points that simulate the positions of the sensors. A dummy of the airplane model described is shown in Fig.1. The purpose is to derive the natural modes of vibration of the structure and the corresponding natural frequencies, assuming different damping conditions and load cases i.e. placing the force in different nodes of the structure.

The different examined cases are:

1. Force placed in node *11* and no damping;
2. Force placed in node *11* and distributed damping ζ=0.02;
3. Force placed in node *61* and distributed damping ζ=0.02.
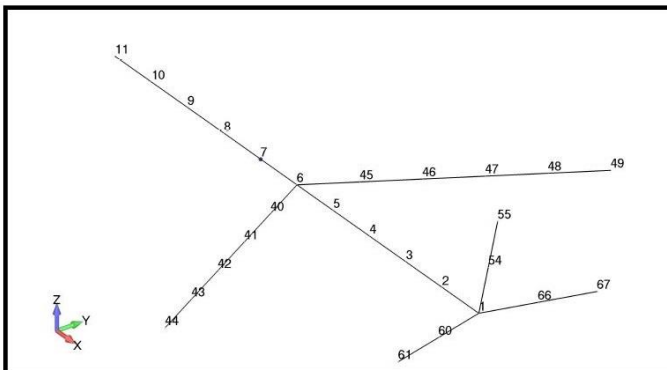
Fig. 1 Dummy aircraft for simulation of EMA with ModLab

CASE 1. In this case the force is applied in node *11* at the nose of the airplane and it is assumed that there is no damping.
The Accelerance curve as a function of frequency is shown in Fig.2. Considering the number of peaks in the graph, we expect to find at least one structure mode.
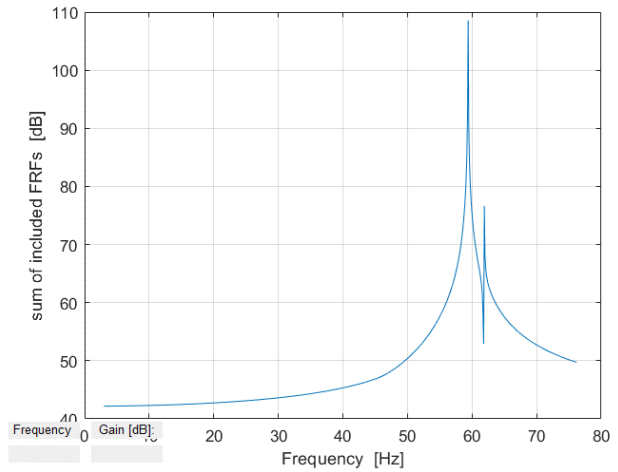

Fig. 2 FRFs plot – case 1

A. Line-Fit

By selecting a subinterval containing the peaks, Line-Fit method extrapolates one structure mode characterized by a natural frequency equal to

$$\omega_n = 59.38 \text{ Hz}$$

A flexural modal shape of the fuselage corresponds to this natural frequency as indicated in Fig.3.
From the analysis of the Bode diagram in Fig.4, it is possible to observe that the estimated curve and the measured curve coincide.
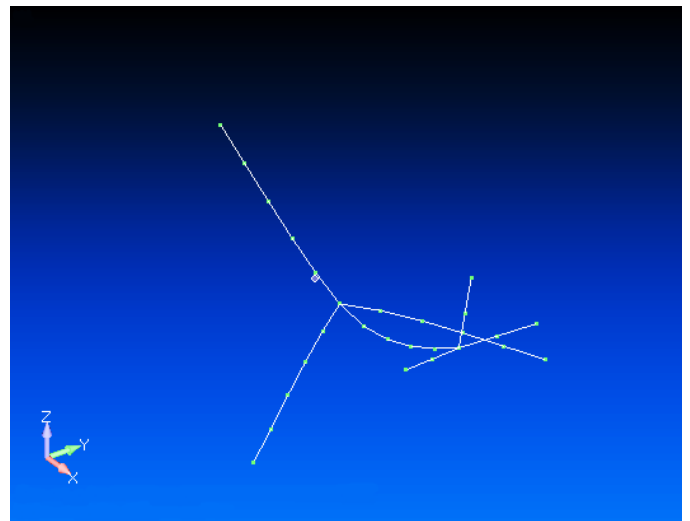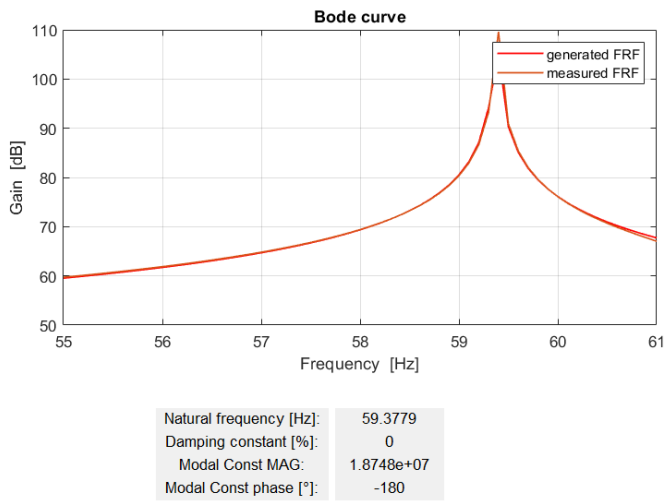

Fig. 3 Flexural modal shape

Natural frequency [Hz]:	59.3779
Damping constant [%]:	0
Modal Const MAG:	1.8748e+07
Modal Const phase [°]:	-180

Fig. 4 Bode diagram and Modal Analysis Results – Line-Fit – case 1

Fig.5 shows the plots obtained in the Dobson's method. The examined case relates to a structure without damping so that the data correspond to a real mode. This explains the horizontal trend without slope of the imaginary part of the graph. If the mode had been complex, the graphs on the right would have been similar to those on the left.
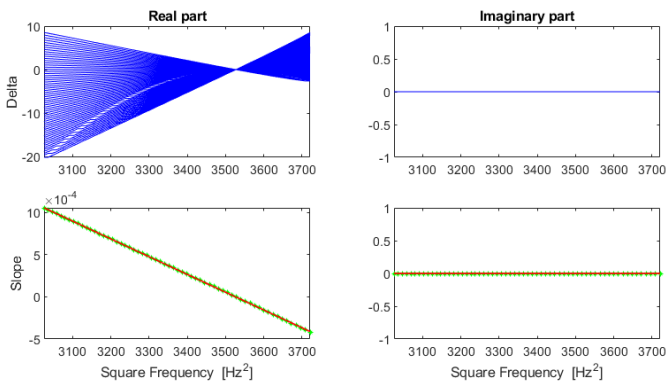
Fig. 5 Dobson's plots – Line-Fit – case 1

B. Circle-Fit

By studying this case it is possible to notice that Nyquist plot is not convenient for use in the analysis of non-damped or lightly damped systems. For Circle-Fit method, circular loop described by the data corresponds to the phase shift suffered by the response relative to the force excitation. This phase shift tends to take place within an increasingly narrower frequency range with increasingly lower values of the damping so Nyquist plot becomes useless when damping is very low.

In Fig. 6 the data points are all concentrated along the axis origin and there is no visible loop on the display.
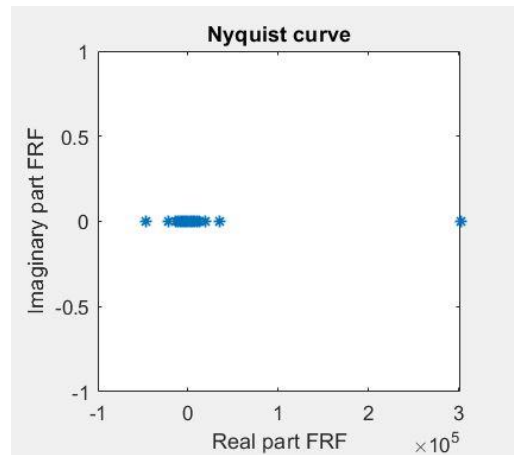
Fig. 6 Nyquist curve for a non-damped structure

C. LSCE

The application of the LSCE method allows to identify one mode shape. From the stabilization diagram it is possible to identify a certain number of probable modes of the structure. However, some of these are fictitious modes and have no physical meaning. Fictitious modes are due, for example, to approximation errors related to residuals. Therefore, from considerations for example about the sign of the damping, it is possible to discern the mathematical modes from the physical one that occurs, to the natural frequency

$$\omega_n = 58.87 \text{ Hz}$$

The stabilization diagram has been obtained using the following values:
   1% for frequency stability;
   5% for damping stability.
   In other words, the iterative process generates a symbol on the graph when the variation over consecutive model orders of the damped natural frequency is smaller than 1% whereas the damping ratios varies with less than 5%.
The stabilization diagram in Fig. 7 confirms the flexural mode identified with the previous parametric identification methods.
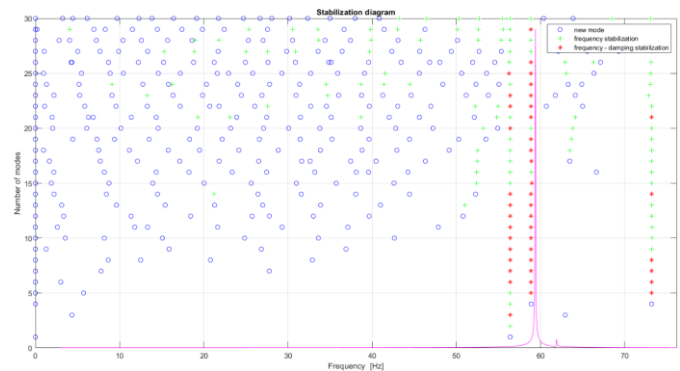

Fig. 7 Stabilization chart with the frequency (+) and frequency and damping (*) stable poles – case 1

CASE 2. Following the introduction of the damping, a reduction of the peak of the FRFs curve is obviously observed in Fig. 8.

The introduction of damping in the system gives a complex form for FRF, containing both magnitude and phase information. Companion plot for Real Part vs Frequency and Imaginary Part vs Frequency are shown in Fig. 9. The phase change through the resonance region is characterised by a sign change in one part accompanied by a peak value in other part.
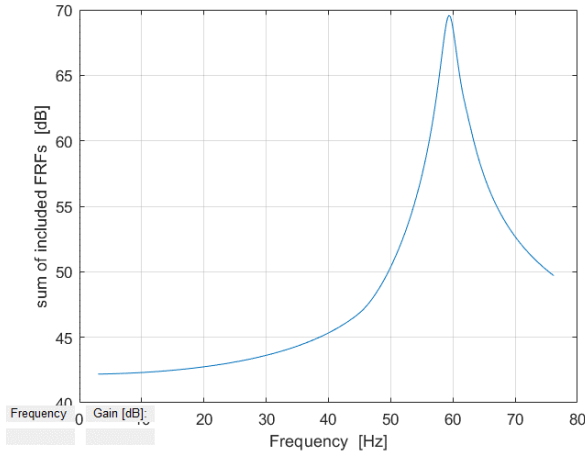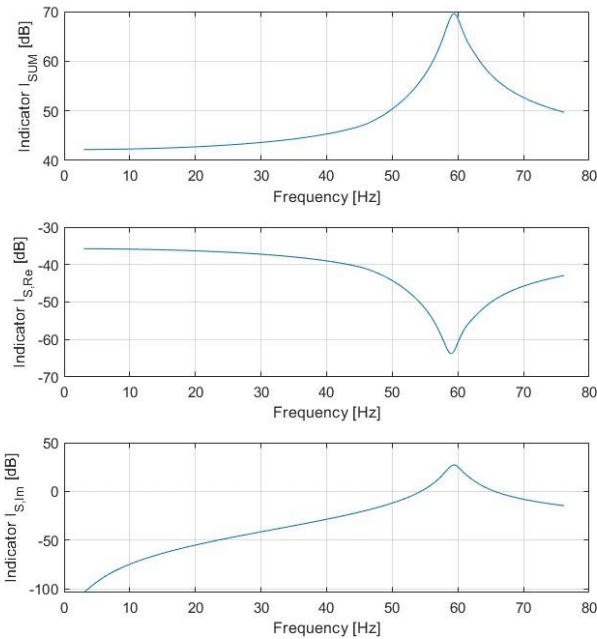

Fig. 8 FRFs Plot – case 2

Fig. 9 FRFs, Real part and Imaginary part – case 2

A. Line-Fit

An increase in damping involves a slightly appreciable increase in the natural frequency, compared to the previous case. The natural frequency and damping are equal to

$$\omega_n = 59.42 \text{ Hz} \qquad \zeta = 2.21 \text{ [%]}$$

Once the modal parameters extracted from the fitting method are known, it is possible to plot a curve based on the results obtained. Observing the Bode diagram and comparing it with

that of the previous case (Fig.4), we can derive the consideration that as the damping increases, not only the peak of the Accelerance decreases, but it becomes more difficult to be able to determine the natural frequency: the iterative process is such that the estimated curve deviates more from the measured one (Fig.10).
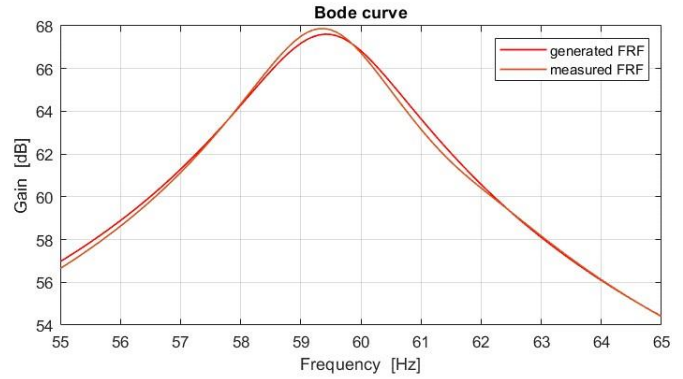
Fig.10 Bode diagram – Line-Fit – case 2
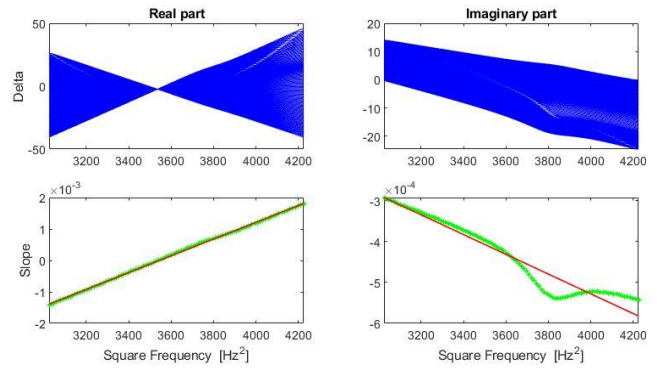
Fig.11 Dobson's plots – Line-Fit – case 2

B. Circle-Fit

Differently from the previous case, in which the structure is devoid of damping, the Circle-fit method can be applied to the case in question. The natural frequency is determined by the maximum angular spacing method where the natural frequency is assumed to occur at the point of maximum rate of change of angle between data points in the complex plane. In other words, the natural frequency is located therefore at the midpoint of the major arc between two frequencies. Natural frequency and damping are

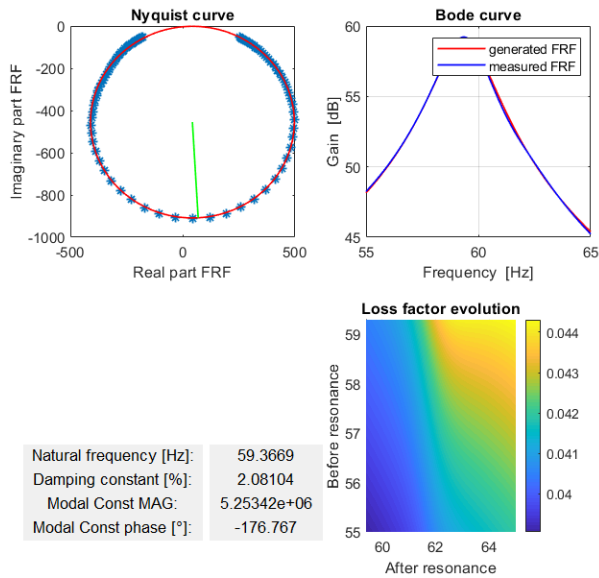$$\omega_n = 59.37 \text{ Hz} \qquad \zeta = 2.08 \text{ [%]}$$

Fig. 12 Nyquist curve, Bode diagram and Loss factor evolution plot – case 2

C. LSCE

Through the application of the LSCE method and through an analysis of the stabilization diagram, ModLab identifies variuos probable modes of the structure. From considerations on the stabilization values, the only one that occurs and that it has physical value, introduces a natural frequency and a damping equal to
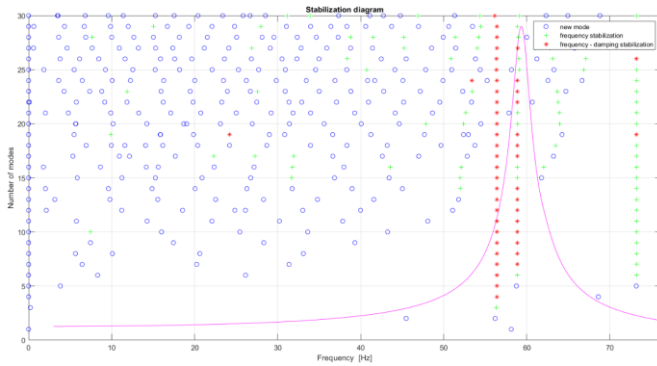
$$\omega_n = 59.11 \text{ Hz} \qquad \zeta = 1.56 \, [\%]$$

Fig. 13 Stabilization chart with the frequency (+) and frequency and damping (*) stable poles – case 2

CASE 3. In the case in which the force is applied in node *61* at the left tip of the horizontal tail of the airplane, considering the number of peaks in the Accelerance plot as a function of frequency in Fig.14, we expect to find two structure modes.
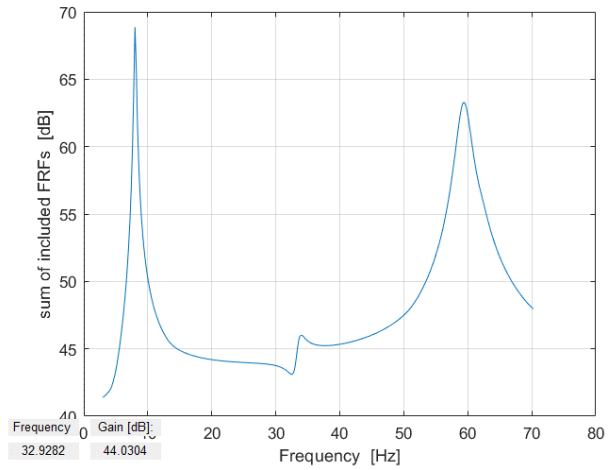
Fig.14 FRFs Plot – case 3
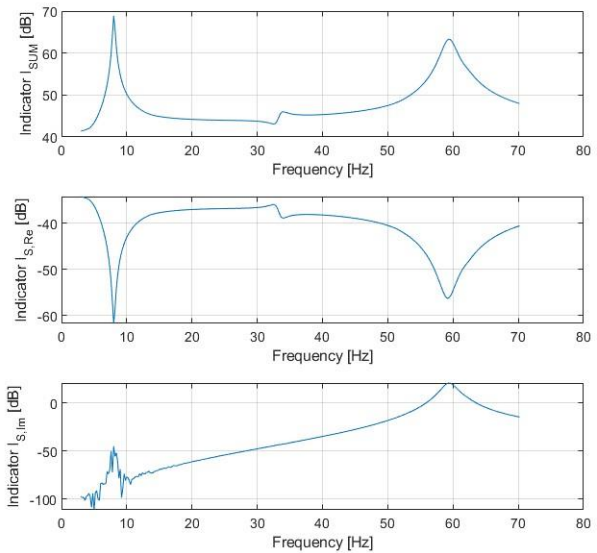
Fig. 15 FRFs, Real part and Imaginary part – case 3

A. Line-Fit

Line-Fit method is able to extrapolate two structure modes characterized by the following values of natural frequency and damping:

$$\omega_{n1} = 8.44 \text{ Hz} \qquad \zeta_1 = 2.00 \, [\%]$$

$$\omega_{n2} = 59.43 \text{ Hz} \qquad \zeta_2 = 2.21 \, [\%]$$

A flexural modal shape of the fuselage corresponds to the more elevated natural frequency (ω_{n2}), already identified in the precedent analyzed cases.

Fig. 16 shows a twisting mode shape corresponding to the lower frequency (ω_{n1}). The fuselage twists and consequently the horizontal tail and the wings are characterized by large deformations.
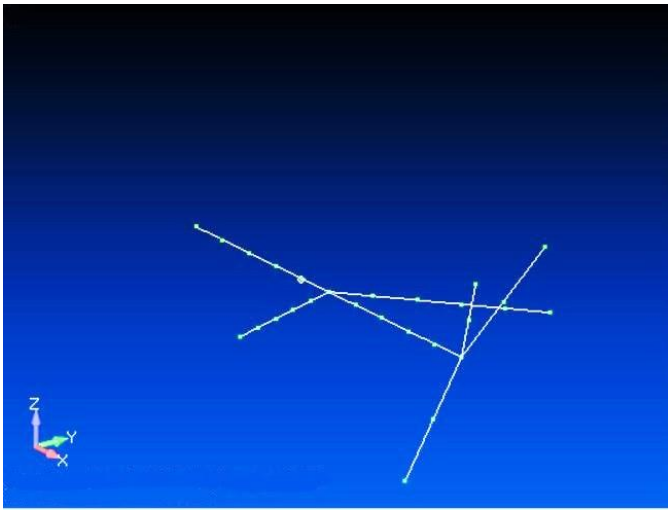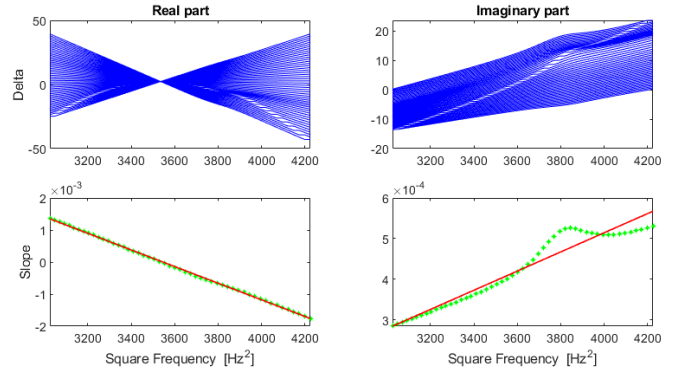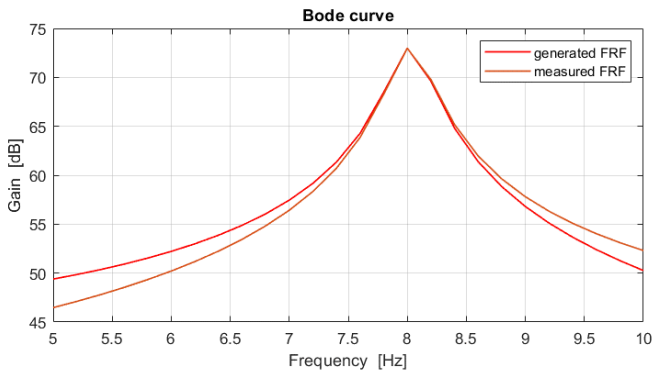
Fig. 16 Twisting mode shape – case 3
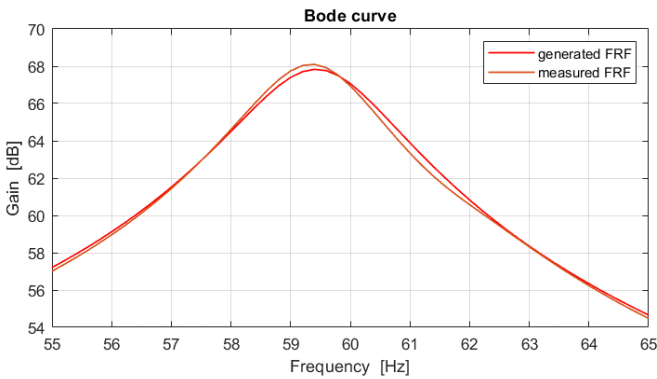

Fig.17 Bode diagram – 1st mode natural frequency – case 3
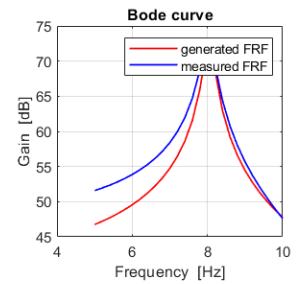
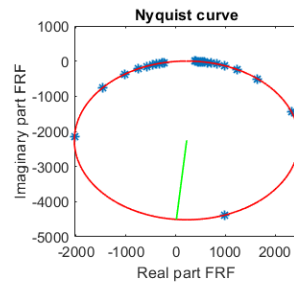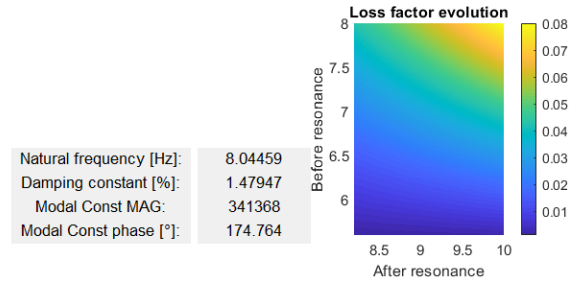Fig. 18 Bode diagram– 2nd mode natural frequency – case 3

Fig. 19 Dobson's plots – 1st mode – case 3

Fig. 20 Dobson's plots – 2nd mode – case 3

B. Circle-Fit

$\omega_{n1} = 8.05$ Hz $\qquad$ $\zeta_1 = 1.48$ [%]

$\omega_{n2} = 59.37$ Hz $\qquad$ $\zeta_2 = 2.12$ [%]

Natural frequency [Hz]:	8.04459
Damping constant [%]:	1.47947
Modal Const MAG:	341368
Modal Const phase [°]:	174.764

Fig. 21 Nyquist curve, Bode diagram and Loss factor evolution plot – 1st mode – case 3
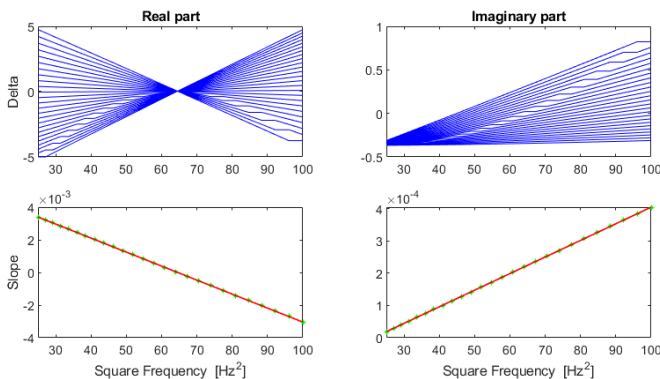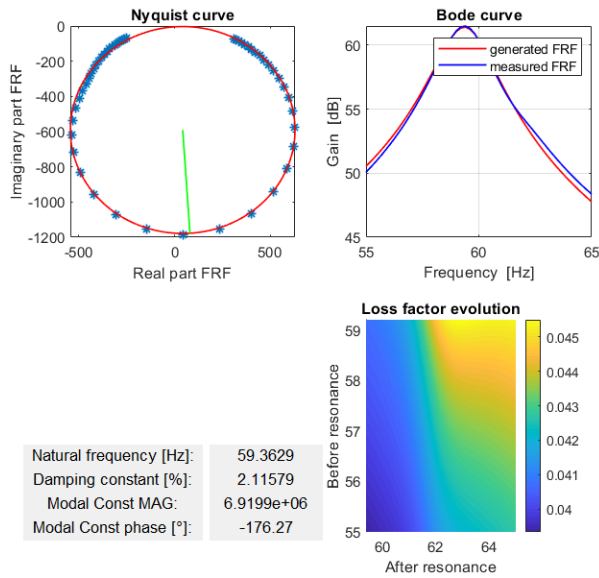
Fig. 22 Nyquist curve, Bode diagram and Loss factor evolution plot – 2nd mode – case 3

C. LSCE

$$\omega_{n1} = 5.03 \text{ Hz} \qquad \zeta_1 = 3.19 \text{ [\%]}$$

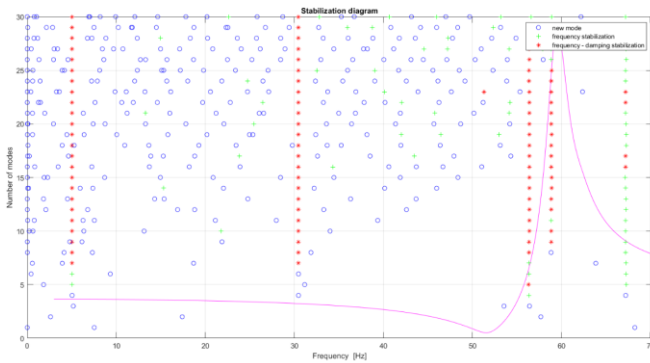$$\omega_{n2} = 58.88 \text{ Hz} \qquad \zeta_2 = 2.07 \text{ [\%]}$$

Fig. 23 Stabilization chart with the frequency (+) and frequency and damping (*) stable poles – case 3

IV. COMPARISON BETWEEN TESTLAB AND MODLAB RESULTS

In order to validate and test ModLab also with real structures, an EMA was computed, and a comparison between ModLab results and TestLab results was made. The modal testing was about a bogie skirt used on a high-speed train. In operational conditions, the bogie skirt is installed in the lower part of the train, in proximity of the bogie, with the aim to reduce or screen both, rolling and aero acoustic noise sources, to the exterior.

The tested components are listed below:

• one fiberglass fairing made with manual layering of polyester resin and glass fiber sheets in biaxial reinforcement and foam core 10 mm thick;

• three locks in stainless steel;

• three support brackets and attachment to the vehicle in carbon steel and with cage nut;

Fig. 24 shows in detail the drawing of the investigated object, from which the main geometrical dimensions can be seen:

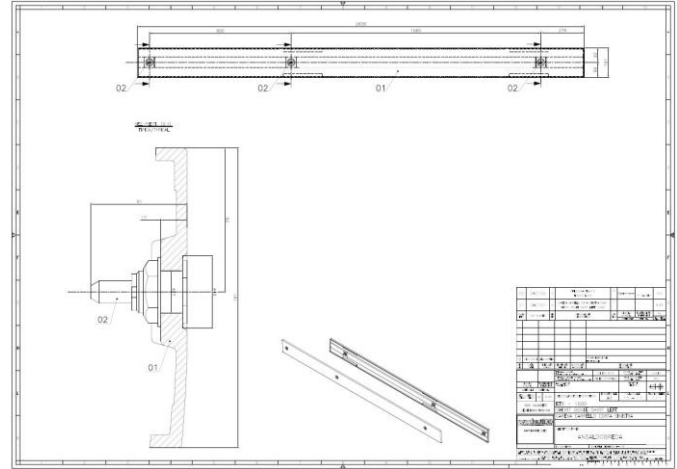• length =2836 mm;

• width = 191 mm.

Fig. 24 Drawing of the bogie skirt

The first step in setting up the skirt for frequency response measurements was to consider the fixturing mechanism necessary to obtain the desired constraints.

Regarding the supporting of the skirt, for testing purposes, the train structure was simulated by means of an aluminium tube with rectangular section, to which the support brackets were connected. The dimensions of the section are 100 x 50 mm, 4 mm thick.

The analysis was performed taking into account the stiffness of the bolts that connect the skirt to the support brackets.

The attachment points have indeed some degree of flexibility. However, the tube can be considered almost fixed in the frequency range of interest, being its acceleration at least one order of magnitude lower than ones measured on the skirt. Thus the response of the base of the attachment had a negligible effect.

About the EMA set-up and measurement procedure, test system configurations consists of:

• Integrated Front-end and Spectral analyser LMS Scadas III, 16 channels;

• Modal hammer ENDEVCO 2302-10;

• 4 accelerometers PCB Model 333B32;

• LMS TestLab software.

Fig. 25 Test Setup of the bogie skirt

Regarding the EMA measurement procedure, before starting the experimental tests, 13 stations were identified along the longitudinal axis of the bogie skirt. The distance between two consecutive stations was taken constant and equal to Δx = 225 mm. For each station 2 measurement positions along the transversal axis were fixed, for a total amount of 26 measurement positions.

Referring to Fig. 27, the force has been input to the structure by the impact hammer individually in each position, while the output has been measured in 4 positions, identified as 21, 61, 91 e 122, by means of 4 single axis ICP accelerometers. This allowed to have a total of 26x4=104 FRF to analyse.

The natural frequencies and damping values extracted for the bogie skirt by means of TestLab are recalled in the Table I.
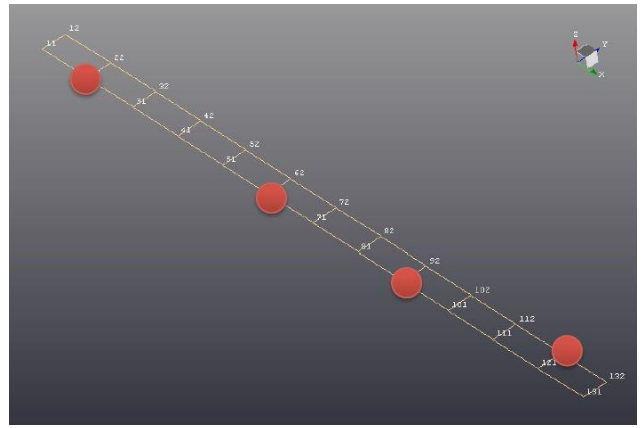

Fig. 26 Modal hammer ENDEVCO 2302-10

Fig. 27 Measurement grid and accelerometers positions

TABLE I
NATURAL FREQUENCIES AND DAMPING - TESTLAB

No.	Frequency [Hz]	Damping [%]
1	22.68	1.75
2	54.74	0.97
3	60.21	1.26
4	68.03	1.29
5	82.89	1.15
6	88.07	1.51

Being able to use LMS TestLab software not only for the data acquisition and analysis but also for the post processing, once determined the values of natural frequencies and damping, we could proceed to plot the corresponding mode shapes. Figs. 28, 29, 30 show the first three mode shapes of the bogie skirt.
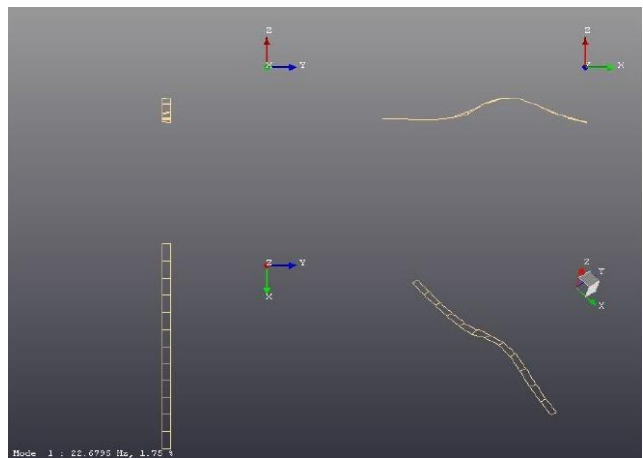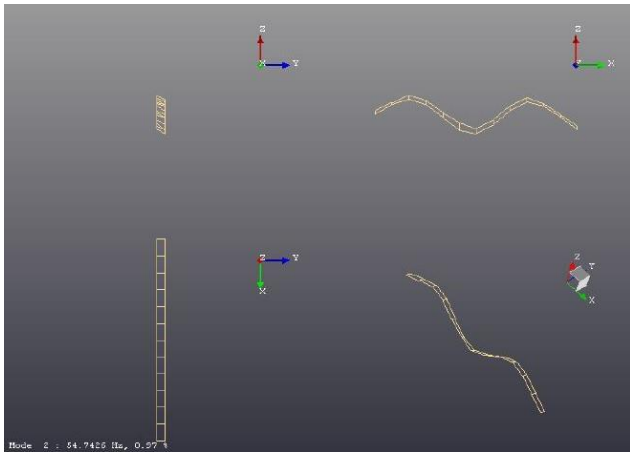

Fig. 28 Mode shape 1 - TestLab

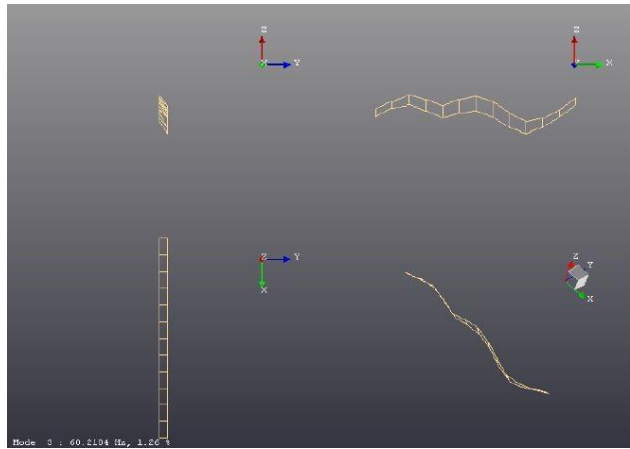Fig. 29 Mode shape 2 – TestLab

Fig. 30 Mode shape 3 - TestLab

The quality of the modal parameters identified can be evaluated through comparison with the results obtained in the previous paragraph. Fig. 31 shows the measured FRFs. It is possible to identify 6 peaks focusing attention on the 0 -100 Hz frequency range. Tables II, III and IV show the results obtained by adopting the three different methods of modal parametric identification implemented by ModLab.
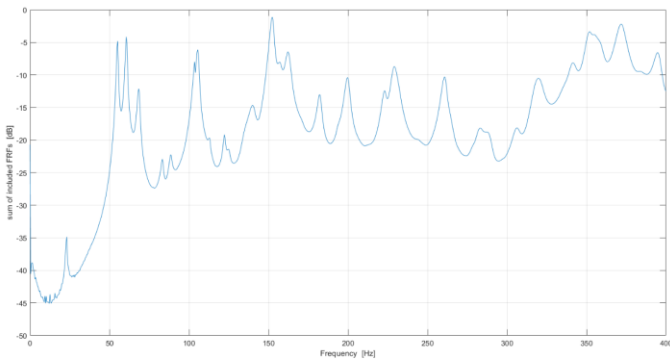

Fig. 31 FRFs Plot

TABLE II
NATURAL FREQUENCIES AND DAMPING – MODLAB - LINEFIT

No.	Frequency [Hz]	Damping [%]
1	22.89	1.62
2	54.84	0.84
3	60.40	1.26
4	68.03	1.24
5	83.38	1.20
6	88.16	1.52

TABLE III
NATURAL FREQ. AND DAMPING – MODLAB - CIRCLEFIT

No.	Frequency [Hz]	Damping [%]
1	23.01	1.68
2	54.99	1.02
3	60.68	1.14
4	68.3	1.16
5	83.79	1.19
6	88.36	1.35

TABLE IV
NATURAL FREQUENCIES AND DAMPING – MODLAB - LSCE

No.	Frequency [Hz]	Damping [%]
1	22.81	1.56
2	55.04	0.86
3	60.24	0.84
4	-------	-------
5	-------	-------
6	88.51	1.31

Fig. 32 Line-fit plots

Fig. 34 LSCE Stabilization chart
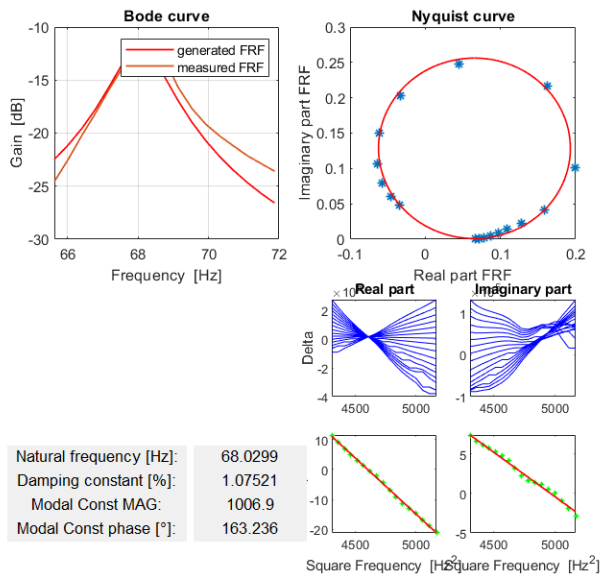
Finally, Table V shows the percentage deviations from the TestLab reference values. Estimated modal parameters from the TestLab and ModLab are presented and compared, for the first six modes of the skirt. The green boxes show a deviation of the results of less than 1%, while the red ones show a deviation of more than 10%. It is found that natural frequencies are almost always deviating by less than 1%, whereas damping results are much less reliable.
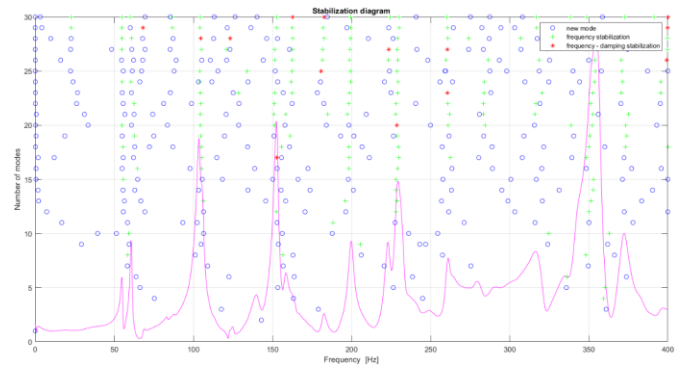
TABLE V
PERCENTAGE DEVIATIONS OF NATURAL FREQUENCIES AND DAMPING FROM THE TESTLAB REFERENCE VALUES

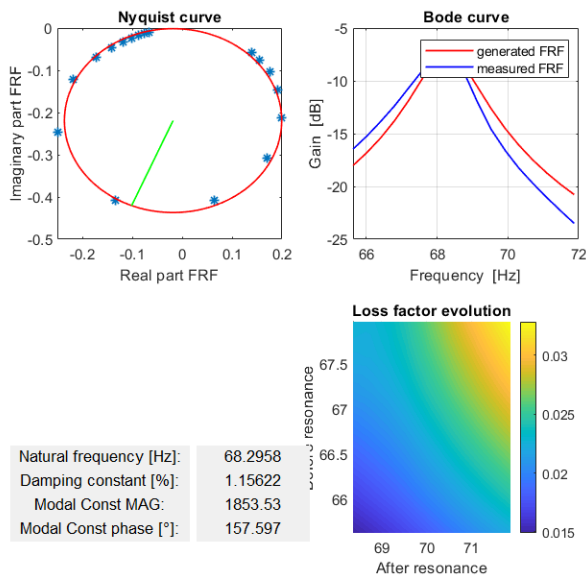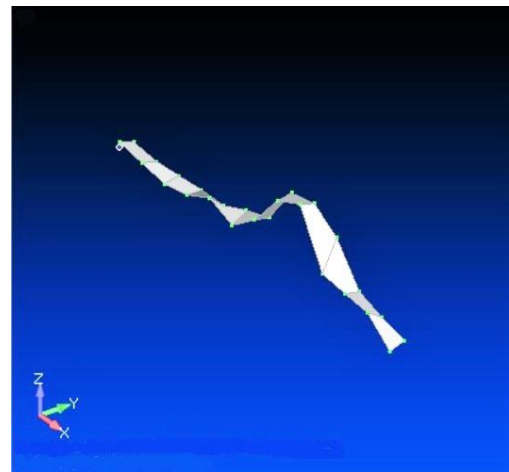| Mode	Line-fit		Circle-fit		LSCE	
	Δf [%]	$\Delta \zeta$ [%]	Δf [%]	$\Delta \zeta$ [%]	Δf [%]	$\Delta \zeta$ [%]
1	0.93	7.43	1.46	4	0.57	10.86
2	0.18	13.4	0.46	5.15	0.55	11.34
3	0.32	0	0.78	9.52	0.05	33.33
4	0	3.88	0.4	10.08	-	-
5	0.59	4.35	1.09	3.48	-	-
6	0.1	0.66	0.33	10.6	0.5	13.25

Fig. 33 Circle-fit plots
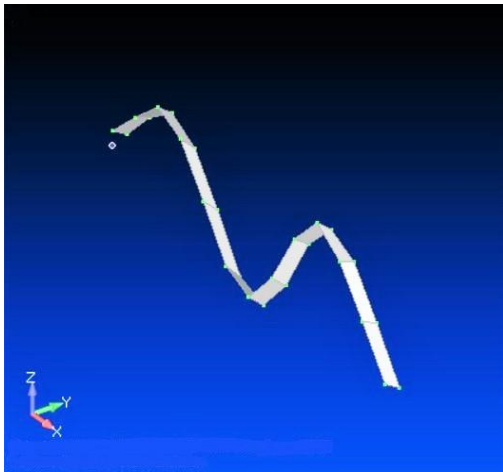

Fig. 35 Mode shape 1 - ModLab

Fig. 36 Mode shape 2 - ModLab

Fig. 37 Mode shape 3 - ModLab

V. CONCLUSION

The simulated and experimental analysis reported in this study were computed in order to test and validate the new Matlab toolbox proposed for the modal parameters estimation into EMA field.

The different simulated case studies have revealed results that are physically verifiable in reality. Therefore, a strong consistency of the results obtained with the different methods of parametric extraction is observed.

As regards the analysis of experimental data, the comparison of modal parameters obtained through ModLab with those obtained through TestLab, has provided percentage deviations lower than 1% in most cases.

Therefore, it is concluded that ModLab can be considered a valid alternative to the most widespread commercial modal analysis software.

A possible future implementation of ModLab could concern the possibility of using the software not only for EMA but also for *Operational Modal Analysis.*

EMA method presumes that the user has some knowledge both about inputs and outputs, but OMA is appropriate for operating machines because it relies only on the measured outputs and it does not need any knowledge about the input forces.

OMA has many advantages compared to EMA. In fact, in traditional EMA an artificial excitation is usually used for measuring the FRF. These quantities would be very difficult or even impossible to be measured in field tests and/or for large structures, such as bridges. Moreover, EMA is typically conducted in a laboratory environment with proper boundary conditions. The dynamic of the system is excited by less realistic conditions than would occur in the real-life environment. OMA technique was developed for the identification of the modal parameters of the structures that in industrial applications show real operating conditions quite different from those applied in the lab experiments. The diffusion of remotely managed sensors and measurement equipment, with a proper Structural Health Monitoring strategy, has allowed the development of an "*On Condition*" preventive maintenance philosophy, of a plant or a component, in order to promptly determine if it could remain in service or should be removed and replaced. Therefore, by using OMA the true mode shapes are obtained when the structure is operating with all the components assembled and the OMA parameter estimation can keep track of the variation of the modal parameters and inform the user when an unforeseen maintenance service requests to be done.

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