Development of a tilt-rotor model for real-time flight simulation

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The aim of this work is to describe the development of a tilt-rotor model for real-time flight simulation based on the NASA XV-15 prototype aircraft, for the integration with the ReDSim Research and Didactic Flight Simulator of the ZAV Center of Aviation, at ZHAW Zurich University of Applied Science in Winterthur, Switzerland. A simulation model is developed in Matlab/Simulink® based on available literature and several off-line tests are carried out for both the helicopter and the airplane modes of the aircraft. Furthermore, the simulation model is integrated with the ReDSim and several pilot tests are performed. Results are presented, weaknesses of the model are evaluated and further developments are suggested. Present work is the result of a Final Thesis submitted as partial fulfillment of requirements for the author's Master's of Science in Aerospace Engineering.

Nomenclature

α	=	aerodynamic angle of attack
β_M	=	rotor mast tilt angle
δ_F	=	flaps angular
λ	=	inflow parameter
λ_0	=	free-stream inflow parameter
ϵ	=	down-wash angle
μ	=	advance ratio
φ,ψ,θ	=	Euler angles
ρ_{AIR}	=	air density
C_l	=	generic lift coefficient
C_d	=	generic drag coefficient
C_m	=	generic moment coefficient
C_T	=	thrust coefficient
c.g	=	abbreviation for center of gravity
g	=	acceleration of gravity
G	=	ground effect factor
h	=	generic discrete time-step
I_{XX}	=	rolling moment of inertia about c.g.
I_{XZ}	=	product of inertia about c.g.
I_{YY}	=	pitching moment of inertia about c.g.
I_{ZZ}	=	yawing moment of inertia about c.g.
l,M,N	=	generic roll, pitch and yaw moments around body axis
M_N	=	Mach number
R	=	subscript for quantities related to rotors
SL,BL,WL	=	generic station-line, butt-line and water-line body coordinates
T_{AIR}	=	air temperature
u	=	generic input
U,V,W	=	generic components of velocity in body axis
X,Y,Z	=	generic components of force in body axis
X_{SF}	=	side-force rotor effect factor
X_{SS}	=	side-by-side rotor effect factor
y	=	generic output

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I. Introduction

FOLLOWING a long series of attempts over the past century, the helicopter has become a product thanks to its unique capabilities which allows it to stand as an unbeatable platform for several military as well as civil roles: as a matter of fact, the helicopter fits particularly well commercial roles such as rescue and heli-ambulance, police support, civil patrol, and survey, isolated site refurbishment, oil and gas platform shuttle, point to point transport, as well as military tasks among which anti-tank, troop carrier, combat search and rescue, maritime patrol and many others. Nevertheless, the rotorcraft typically carries several, strong limitations on speed, range, noise and comfort of passengers.
In a scenario where innovation appears as fundamental, the tilt-rotor takes its chances to establish a new standard in the aviation world. On a tilt-rotor aircraft, rotors can either produce lift in helicopter mode or thrust in airplane mode (lift is then provided by the wing), yaw control is provided without requiring an anti-torque system. Moreover, thanks to its conversion capabilities which indeed stand as a breakthrough technology, the tilt-rotor aircraft can provide high productivity, high speed, longer range and high versatility. On the other hand, the tilt-rotor is far from being the perfect solution, since its estimated cost is currently higher than that of helicopters and several additional operating and maintenance costs shall be considered due to its mechanical complexity and the coexistence of both aircraft and helicopter designs. Moreover, current tilt-rotors have higher rotor disk loading and lower power loading than conventional helicopters, aspects meaning worse hovering capability. Nevertheless, tilt-rotors are meant to fulfil quite well all those missions requiring both vertical lift and high speed capabilities. Given the general academic and industrial interest in tilt-rotors, the need for Research & Development simulators of such a platform appears rather relevant.

II. Tilt-rotor Mathematical Model

The mathematical model is mainly derived from ref. [1], which is the result of the development of a flight simulation model for the XV-15 Research Tilt-Rotor Aircraft, a joint project of NASA and Bell Helicopters carried out during the 70s with the aim of improving tilt-rotor technology and eventually yielding to the development of modern tilt-rotor aircraft such as V-22 Osprey and BA609[2]. The cited report represents the last (previous efforts were made, the most relevant is ref. [3]), complete report available on public domain and it gathers the essential equation suitable for describing the flight dynamics of a tilt-rotor behaviour: the mathematical model is basically a mix of both theoretical, widely accepted models and semi-empiric ones mainly derived from aerodynamic tests in wind tunnels. Nevertheless, if [1] is deeply investigated, it might appear as evident that several data were omitted and the overall software architecture is missing. As a result, the total understanding of the model requires consistent efforts in terms of both time and intellectual commitment since the need for reviews and corrections of errors were indeed not easy to be detected and operated. All things considered, the very report stands as the best document from which to start and build up the new model. For the sake of legibility and since the extensive but limited changes are made from the reference, only a few specific aspects of the mathematical model are discussed in this section, so the reader shall refer to [1] for a deep understanding of the full mathematical model.

A. Rotor Model

The Rotor model is mainly derived from the classical combination of Blade Element Theory[4] and Momentum Theory[5], but is expressed in mast-axis system and introduces flapping dynamics and previsions for prop-rotor characteristics such as non-linear twist flapping restraint and pitch-flap coupling (in short, the model merges the fundamentals of the Blade Element Theory with those of the Actuator Disk Theory and the Momentum Theory). The model is based on several, main assumptions:

1) average values for the lift-curve slope and profile-drag coefficient are assumed as equal over the entire blade span and then adjusted empirically when approximating the rotor thrust and power-required characteristics;
2) small angles approximation is used for blade's angles of attack;
3) harmonics of flapping greater than one-per-revolution are neglected (usual assumption when performing Multi Blade Coordinate Transformation to a three-bladed rotor;
4) blade stall and compressibility effects are approximated by limiting the maximum rotor thrust coefficient as a function of advance ratio.

However, equations were customised by introducing many semi-empirical correction factors and in order to best fit the general mathematical model to the experimental data. Typically, if Blade Element Momentum Theory is used, thrust coefficient and inflow velocity are correlated by an implicit relationship up to the forth order on non-linearity, with no simple algebraic solution. Recalling [1],

$$\lambda = \lambda_0 + \frac{C_T\left[1-(1-G)\left(1+X_{SS}+X_{SF}\right)\right]}{\sqrt{\mu^2+0.866\lambda^2}+\frac{0.6\sqrt[3]{C_T^2}}{|C_T|+8\mu^2}\frac{\left(|C_T|+\frac{8}{3}\lambda|\lambda|\right)}{(|C_T|)+8\lambda^2}} \tag{1}$$

As a matter of fact, the problem can be solved using several numerical methods (e.g. Newton-Raphson and Bisection) or iterative loops[6]. What described in [1] is not fully clear and first stand-alone tests on the deriving algorithm do not show repetitive convergence of the method for all initial conditions. Furthermore, the algorithm showed two nested loops (the one for the inflow and the one for the thrust) which often and alternatively exceeded the maximum number of iterations. Consequently, several attempts were made to modify the algorithm and eventually an improved version is implemented, consisting of a single main iterative loop which is currently ensuring convergence in all tested conditions. At the current stage, the model does not take a dynamic inflow behaviour into account, but further developments may well lead to the introduction of such an improvement. As peculiar of the tilt-rotor aircraft, rotors affect the aerodynamic behaviour of the overall aircraft. Such a contribution appears rather relevant since the wing exerts a Z-force which acts in opposition to the rotor thrust. Moreover, the exerted force has an arm which generates mainly a pitching moment around the center of gravity of the aircraft. As a consequence, at each time step, the model shall compute the wake radius: a simplified exponential relation such that in [1] can be adopted by introducing correction factors to fit the experimental data available. Consequently, the following is computed:

1) the area of the wing affected by the rotor wake is computed as a function of wake radius, tilt angle, wake angle of attack and side-slip angle of the fuselage;
2) the distances between the load point considered for the application of force and the centre of gravity of the aircraft.

Calculation is performed using a rather extended algorithm consisting in [1].

B. Wing-pylon

The main contribution to the aerodynamics of the tilt-rotor aircraft is the so called wing-pylon. Precisely, the following, main effects shall be considered:

1) rotor wake;
2) free-stream airflow;
3) additional drag terms due to the interference between the wing and both pylons as well as secondary elements.

Several modifications are introduced in order to avoid all divisions by zero during a simulation. Eventually, the aerodynamic Database is used to build multidimensional look-up tables as shown in fig. 1

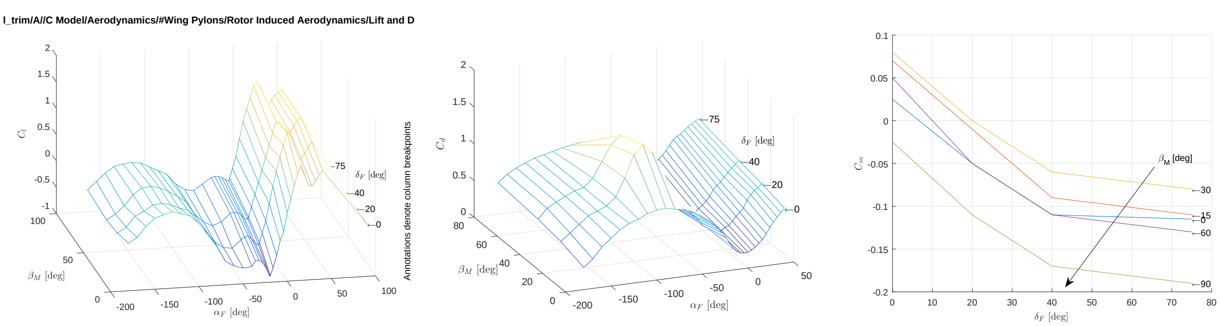

Fig. 1 XV-15 aerodynamic coefficients for the wing pylon (ref [1])

C. Fuselage

The equations for the fuselage model exploits a wide aerodynamic database accounting for the full angular range of motion of the aircraft in terms of both angle of attack and side-slip angle. Precisely:

1) the model neglects the rotor wakes effects on the fuselage aerodynamic forces
2) aero-data for angles of attack and side-slip less than or equal to 20 deg based on wind-tunnel data
3) for angles of attack greater than 20 deg, coefficients are approximated.

D. Tail

The mathematical model of the horizontal and vertical tail is rather simple [1] and no major modifications are needed. Aerodynamic characteristics and flight surfaces parameters are derived by wind-tunnel tests as reported in [1]; look-up tables of the aerodynamic coefficients are built accordingly and shall be considered for both the horizontal and the vertical surfaces (fig. 2). What the model mainly takes into account is:

1) local dynamic pressure calculation;
2) local angle of attack calculation;
3) wing-body blockage;
4) mast angle conversion effect;
5) wing-pylon wake influence;
6) rotor wake influence;
7) angular rates contribution;

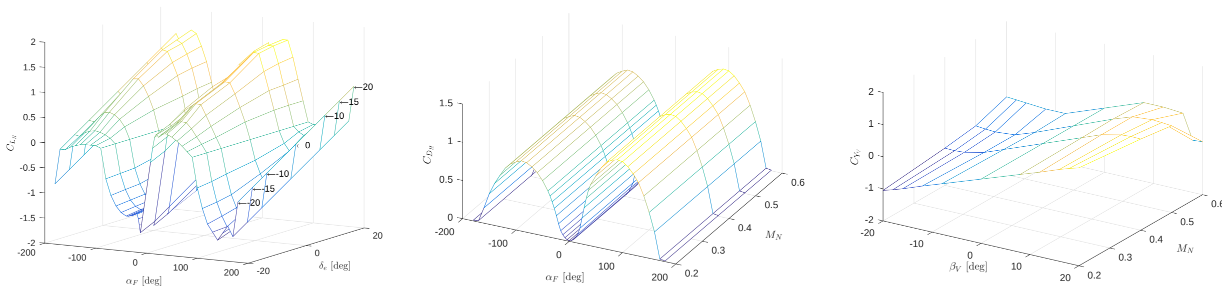

Fig. 2 XV-15 aerodynamic coefficients for the horizontal and vertical tail (ref. [1])

III. Software Architecture

The simulation model is developed in Matlab/Simulink® , an environment which presents several benefits in terms of hardware integration, engineering clarity of the model and debug time: Simulink® allows non-coding experts to implement a simulation model quite easily since most of the coding can be avoided and replaced with functional blocks and small functions which are easier to read, so that the model itself can be managed quickly when implementing modifications as well as during debug. Simulink® also provides the user with the possibility of implementing C-based functions, an option which might be useful in several occasions such as hardware integration. Furthermore, the RedSim (in which the model is meant to be implemented) uses Matlab/Simulink® environment. A top-level snapshot of the Simulink® Model is shown below

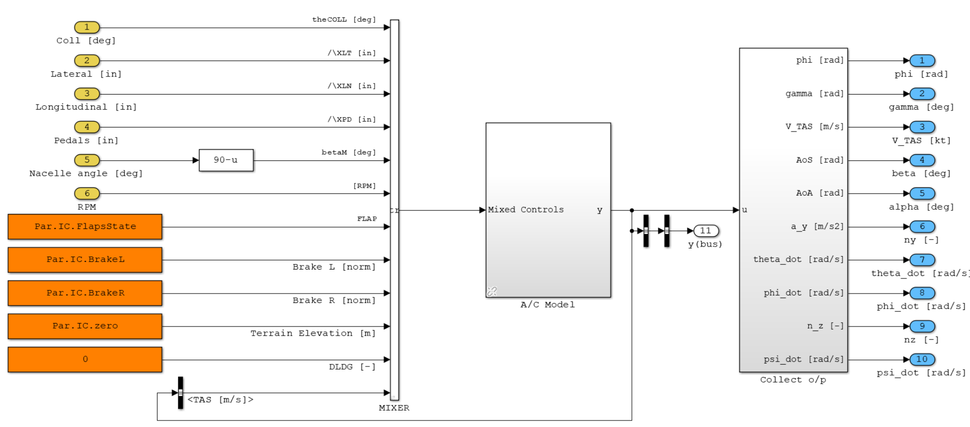

Fig. 3 Top level view of the aircraft model

A. Time-Discrete Simulation

Given the task at hand, the simulation is meant to be time-discrete with constant time-step, therefore time is supposed to move forward in steps of equal duration and the simulator solves model equations successively. As a result, discrete-time-step solvers must be implemented. The model in analysis uses simple Euler discrete integrator, for which the explicit formulation is considered:

$$y_{n+1} = y_n + h \cdot f(t_n, y_n) \tag{2}$$

A simple Simulink® model of a time-discrete, fixed-step integrator can be developed (fig. 4) allowing the connection between discrete-time model and continuous real time.

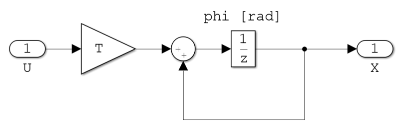

Fig. 4 Euler Explicit integration in Simulink®

B. Aircraft Algorithm

The whole architecture is designed both to comply to ZAV standards and to distribute complexity vertically within levels so that each level of the Simulink® model appears quite simple and easy to understand. As a matter of fact, such a target is to be considered only as partially achieved and several improvements shall be made. The top level architecture of the aircraft model is organized as follows (fig. 5):

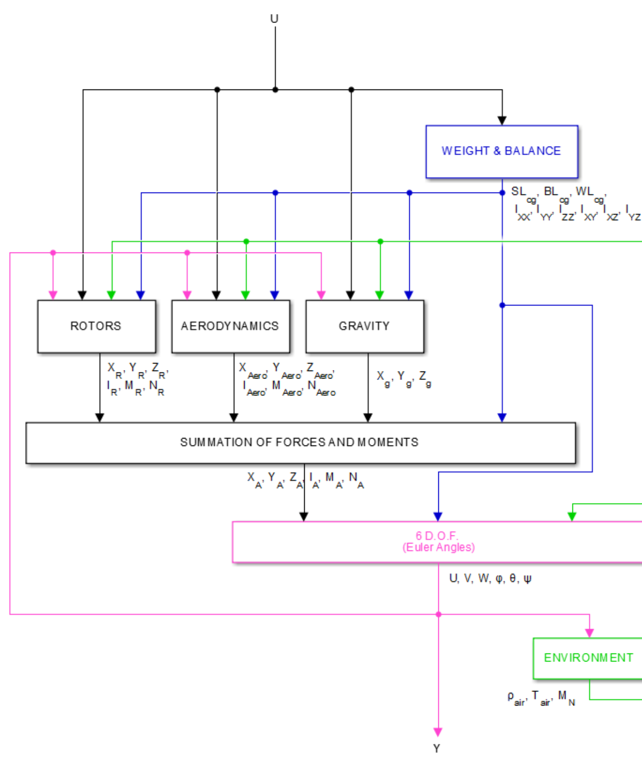

Fig. 5 Aircraft Simulation Block Algorithm

1) A functional subsystem called *weight & balance* gets controls as input signals and computes any displacement from the initial condition for both values of moments of inertia and position of the center of gravity of the aircraft, basically due to the nacelle conversion pilot control;
2) values from *weight & balance* are fed through three subsystems calculating forces and moments of each aircraft main element, basically divided into *rotors*, *aerodynamics* and *gravity*;
3) all forces and moments are then summed in another subsystem called *summation of forces and moments* which provides all components of forces and moments along and about the three, main body axis of the aircraft;

C. Rotor Algorithm
The rotor model is organized as follows (fig. 6) and is meant to be the same for both right and left rotors:
1) (*to wind-mast axis*) block computes coordinates transformation from body axis to local wind-mast axis;
2) all *aerodynamic coefficients* needed for the calculation of the rotor's forces are computed;
3) a third block called *inflow-thrust loop* and performs the iterative procedure used to solve the inflow-thrust non-linear equation;
4) *flap dynamics* block solves a system of 1-st order differential equations modeling the flapping of the rotor[1];
5) as a consequence, the overall forces and moments exerted by the rotor can be determined along and around the three local axis (*forces calculation*);
6) eventually, the resultant forces and moments are transformed back *to body axis*;

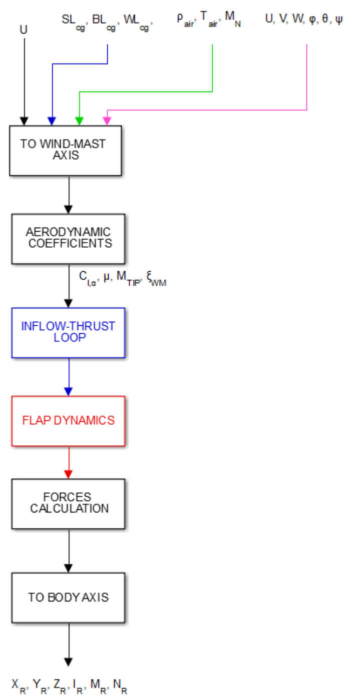

Fig. 6 Rotor Simulation Block Algorithm

D. Aerodynamics Algorithm
The Aerodynamics block accounts for all those elements of the aircraft (apart from the rotors) that exert some kind of aerodynamic force:
1) fuselage;
2) wing-pylon;
3) horizontal tail;
4) vertical tail.

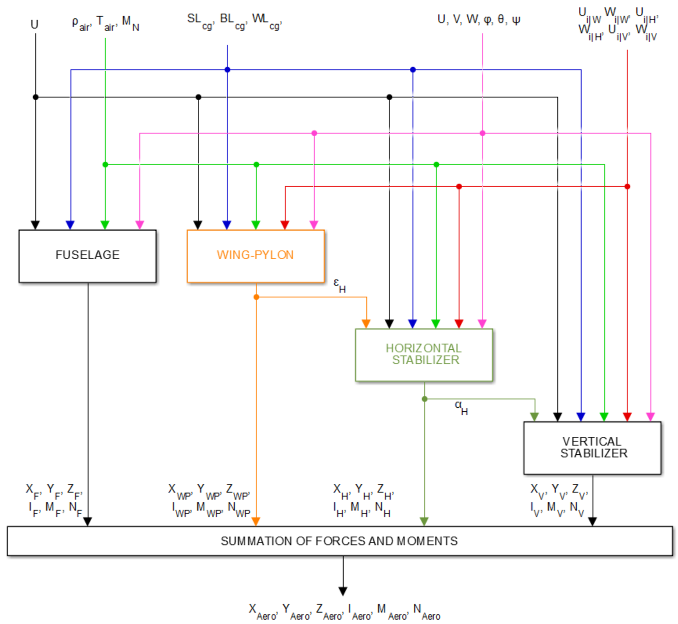

Fig. 7 Aerodynamics Simulation Block Algorithm

The architecture is meant to replicate that of the top level of the aircraft simulation block, so each subsystem is resolved separately and then a further block computes the summation forces and moments; some blocks are depending from one another, so *wing-pylon* computes the value for the down-wash angle, which is then fed through *horizontal stabilizer*; in addiction, *horizontal stabilizer* computes the value for the local angle of attack, which is then taken as an input by *vertical stabilizer*.

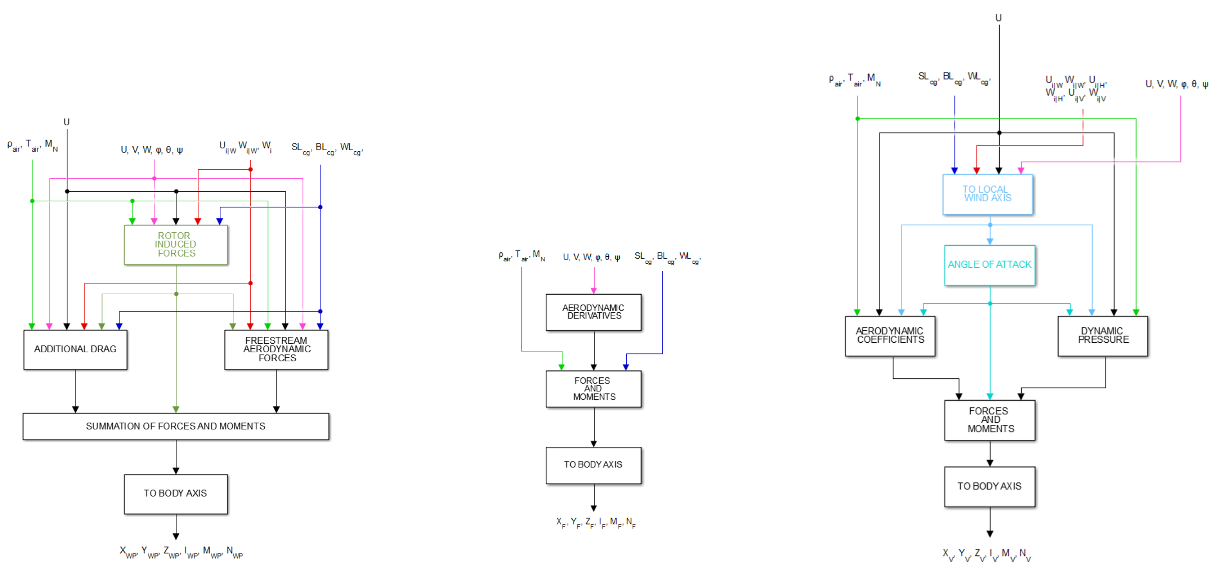

Fig. 8 Simulation Sub-Blocks Algorithms. From the right side: Wing-Pylon, Fuselage, Vertical Stabilizer (the architecture of the Horizontal Stabilizer Block is the same of that of the Vertical Stabilizer)

IV. Preliminary Off-Line Tests

Off-line tests are performed in order to detect hidden errors within the model. As a consequence, both the model and the equations were revised several times in order to assure all symmetries in terms of forces and moments in both the longitudinal and lateral-directional planes are as expected for the aircraft in analysis.

A. Trim

The trim of the ReDSim model is operated using a Matlab®-based routine called trimac.m which shall be interfaced correctly with the Simulink® model. The aircraft model is linked as a library to a Simulink® Model called `model_trim.mdl` which tunes input signals and states of the model achieve a trim condition: all the input and output ports as well as the states of the model shall be named according to what written in `trimac.m`. Trim parameters can be set by editing `TrmLinSim.dat`, a file in which all parameters concerning the trim, linearization and simulation routine can be tuned by the user[7]. Eventually, `trimac.m` collects all parameters as edited in `TrmLinSim.dat` then tunes inputs and states of the model accordingly until the convergence of the Newton-Rapson based algorithm is reached. The straight and level flight condition is considered for trimming the aircraft model. Once a trim condition was achieved this is investigated repeatedly in order to understand whether it is reasonable and realistic or not. As a first evaluation, the very attempt shall be considered as successful enough if all states and output values are reasonable and all inputs are within the acceptable ranges. For instance, input control values are investigated aiming to assess the following:

- trim values for root collective pitch in helicopter mode shall be smaller the greater is the advance speed;
- trim values for longitudinal input in helicopter mode shall increase with advance speed;

As a consequence, a trim condition for the model is searched for helicopter mode at different speeds in order to assure that the expected behaviour is obtained (fig. 9).

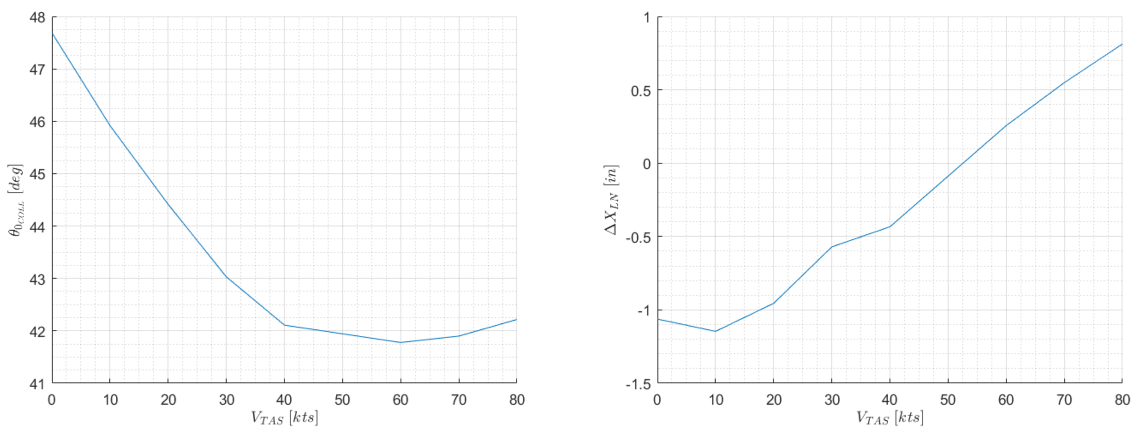

Fig. 9 Root Collective Pitch and Longitudinal Stick Inputs vs Speed

B. Eigenvalues Analysis

Once a trim condition is found for both helicopter and airplane mode, the model can be linearised and the eigen values of the resulting state-space system can be computed. Analyzing the eigenvalues of the linearised model can be rather helpful during debug, since a wrong sign on a stability derivative or a wrong conversion from radians to degrees might determine an unreal instability. Furthermore, as for a preliminary integration of model with the ReDSim a reasonable, trim condition is needed so that the model could be implemented on the flight simulator, tested by pilots and any unexpected behaviour of the aircraft might be highlighted during real-time flight tests. Thanks to eigenvalues analysis the model is revised until two stable, trim conditions are achieved at 70 knots TAS (fig. 10) in helicopter mode and 200 knots TAS (fig. 11) in airplane mode and the model can eventually be implemented in the ReDSim for further tests.

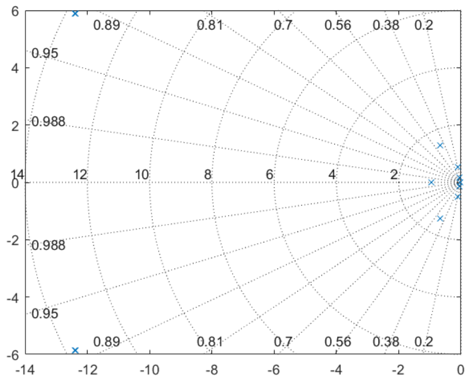

Fig. 10 Eigenvalues in helicopter mode at 70 knots (TAS)

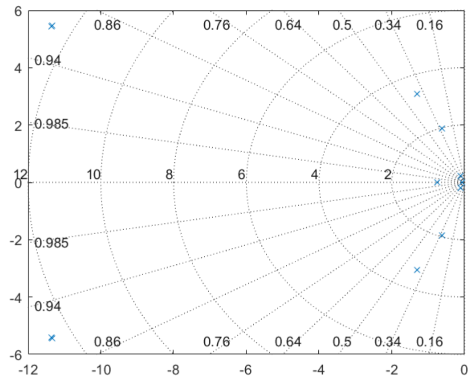

Fig. 11 Eigenvalues in airplane mode at 200 knots (TAS)

C. Off-Line Time Simulations

A few brief off-line simulations is carried out before implementing the model in the flight simulator in order to ensure what expressed by the eigenvalues of the linear system corresponds to what actually shown by the simulation. Results evaluate what previously said (fig. 13, 12, 15, 14).

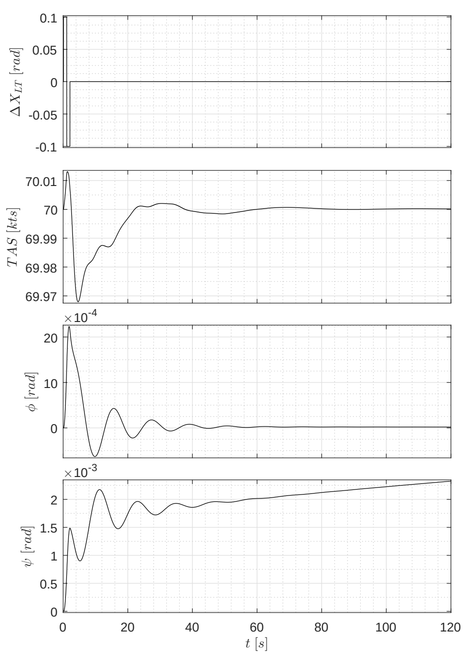

Fig. 12 Time-Response to a doublet lateral input from trim condition in helicopter mode at 70 kts

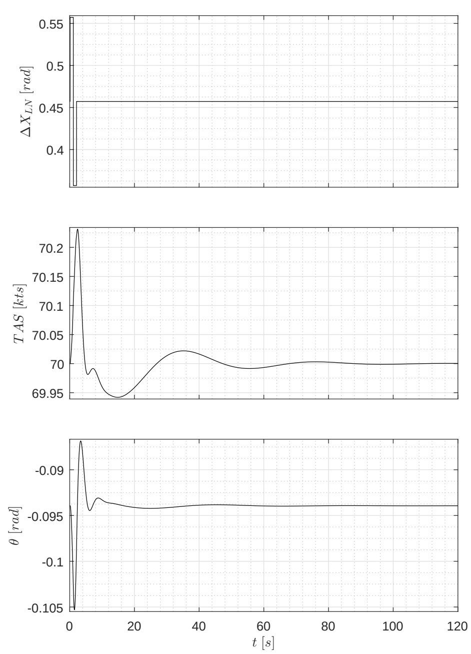

Fig. 13 Time-Response to a doublet longitudinal input from trim condition in helicopter mode at 70 kts

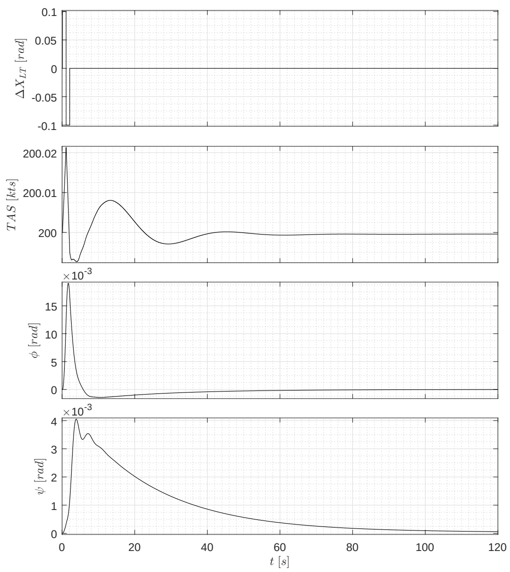

Fig. 14 Time-Response to a doublet lateral input from trim condition in airplane mode at 200 kts

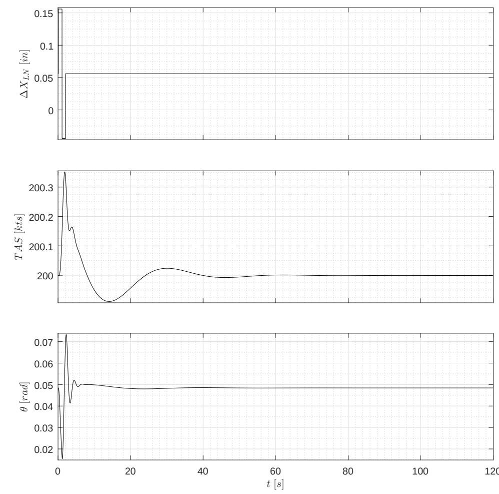

Fig. 15 Time-Response to a doublet longitudinal input from trim condition in airplane mode at 200 kts

V. ReDSim Simulation

The Research and Didactics Simulator ReDSim of the ZHAW is a flight simulator developed the ZAV Center of Aviation and operative since March 2011. The flight simulator is highly realistic thanks to the cockpit-like internal layout, a control loading system which allows to simulate a variety of feedback feel forces on pilot control and a visual system with a field of view of 180 degrees. ReDSim is used for educational activities, research as well as for industrial purposes together with partner companies and it is meant to be a universal platform allowing the interface with a wide range of aircraft models going from conventional fixed-wing airplanes, gliders, UAV to rotorcraft (fig. 16).

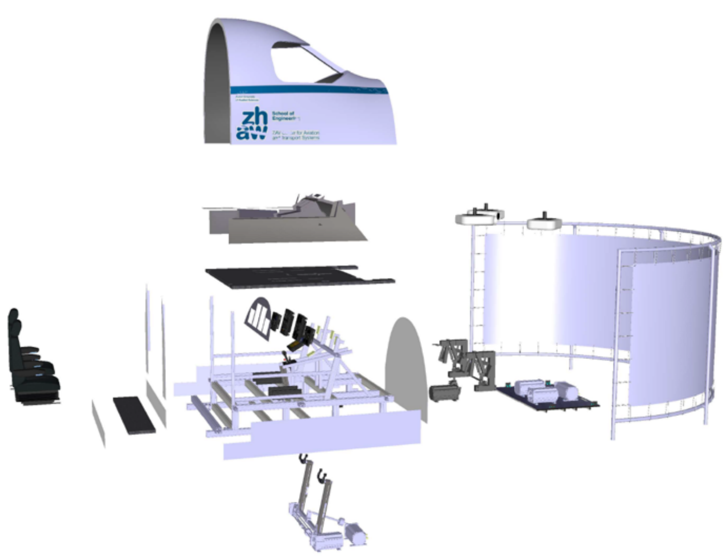

Fig. 16 View of main elements of the ReDSim. Courtesy of ZHAW

A. Integration of the model with the ReDSim

The ReDSim is usually configured for airplanes simulation so the implemented pilot controls are those of a conventional aircraft, meaning:

- yoke control for roll and pitch controls;
- pedals for yaw control;
- throttle for thrust control.

When a tilt-rotor is considered, instead, controls shall be comparable to those of a helicopter, therefore:

- central stick with 2 rotational DOFs, controlling roll and pitch;
- pedals for yaw control;
- collective lever for thrust (combined with engine power lever);

Several modifications on the platform are arried out in order to best adapt it to the tilt-rotor case: the yoke control is removed and replaced with a centre stick, pedals are maintained (fig. 17) On the other hand, replacing the throttle system is currently not possible due to technical and operational issues so the control system is modified so that the throttle could be directly used as collective lever (this solution is meant to be temporary, proper inceptors will be soon implemented).

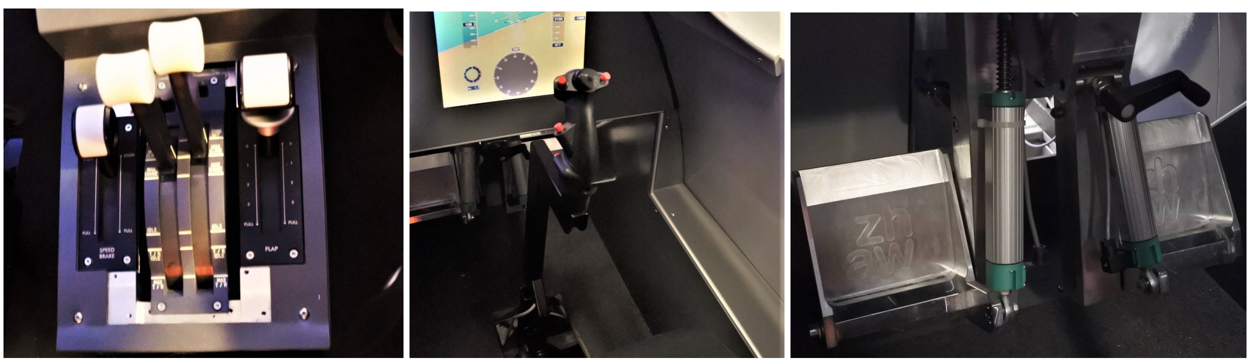

Fig. 17 Pictures showing current ReDSim inceptors configuration

ReDSim integrates a Control Loading System that manages feedback forces to the pilot controls in order to simulate a realistic feel: the Simulation Model get as inputs the force feedback from the actuator, the position feedback from a sensor and the status of the aircraft model (equations of motion), providing the proper servo-valve command signal. The single channels for each control force is revised by taking two main aspects into account, the displacement of the pilot control from neutral position and the overall dynamic pressure acting on the aircraft. As a result gradients of motion of the centre stick and the pedals are read by the model and translated into a proportional feedback force. For the task at hand, a simple model provides a slight, gradient feedback to the pilot as generally accepted for helicopters.

VI. Pilot Tests Result

Given the peculiar nature of the tilt-rotor, the fact that such an aircraft is far from being common within commercial aviation as well as the relatively short notice, preliminary assessment are performed by both an airplane and a helicopter pilot in order to simulate both main flight modes of the XV-15.

A. Helicopter Mode

The pilot is asked to perform:

1) a free, preliminary flight in order familiarize with the model;
2) a few 180° turns at constant altitude;
3) a series of speed captures, in other words to increase forward speed during a series of acceleration laps;

4) a deceleration to zero forward speed in order to reach the hovering condition.

Simulation data records are reported below for both speed capture maneuvers (fig. 18) and deceleration to hover.

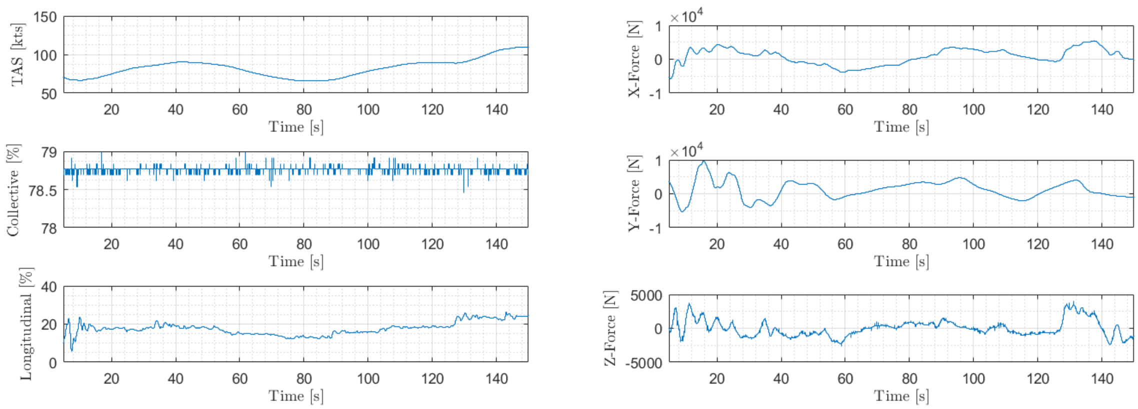

Fig. 18 Real-time pilot-in-the-loop simulation records of a speed capture (helicopter mode)

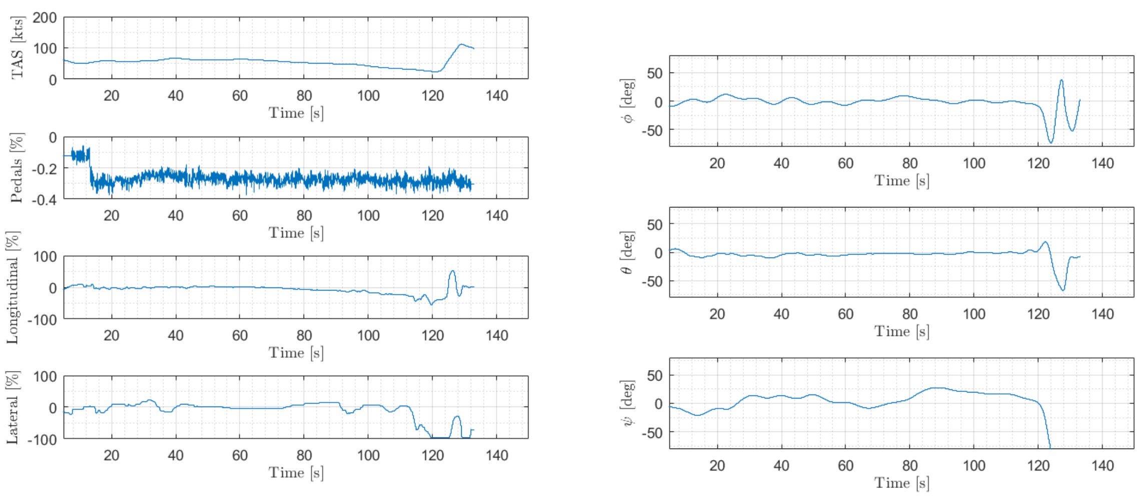

Fig. 19 Real-time pilot-in-the-loop simulation records of a deceleration to hover (helicopter mode)

The model behaves quite smoothly during speed captures attempts (fig. 18) and free flights at high speed in helicopter mode. Nevertheless, as evident in fig.19, tests highlights severe anomalies in the model's behaviour in really low speed flight conditions highlighting the impossibility to reach hovering condition:

1) pilot starts the simulation from TAS 80 kts and then slowly decelerates;
2) the deceleration appears to be performed quite nicely and all parameters concerning the aircraft inertial position follow quite a smooth, realistic trend;
3) when the speed gets closer to zero the aircraft engages strong, unexpected oscillations and the pilot looses authority on the lateral control which appears to be totally ineffective when reached the full, right, position;
4) eventually, the aircraft starts rolling and yawing beyond what expected until heading, attitude and speed are totally lost.

B. Airplane Mode

The pilot is asked to perform:

1) free, preliminary flight in order to gain better familiarity with the model;
2) a few 180° coordinate turns at different bank angles;
3) a series of speed captures;
4) climb at different attitudes.

The general behaviour of the model in aircraft mode appears nice and realistic, the vehicle is controllable as basic navigation can be performed. Nevertheless, a few critical aspects shall be solved with further improvements:

1) full throttle climb rate seams not realistic (drag data used in the model of the whole aircraft shall be improved);
2) aileron control sensitivity is too high;
3) pedal control sensitivity is poor;

Unfortunately, data records from the simulator are not available aircraft mode testing, so results shall be referred to the pilot test card as reported (fig. 20).

Engineering Card	**Project**		**Simulation Number**
	XV-15 Model Development		1
Prepared by	**Test Pilot**		**Aircraft Model**
F. Barra	Sandro Huber		XV-15
Headline	**Date**		**Inflight Card recording**
	14.05.2018		F. Barra
Aircraft weight			
Empty weight	10083 lbs	**Flight Mode**	Airplane
Test pilot weight	185 lbs	**Maximum take-off weight**	13000 lbs
Take-off weight	13000 lbs		
Times			
Flight Duration	0h36m11s	**Initial Condition**	In-flight, 9840 ft
Items		**Time**	**Notes**
180° Coordinate Turn, 20° Bank, TAS 170 kts		n/a	1
180° Coordinate Turn, 45° Bank, TAS 170 kts		n/a	2
Speed intercept TAS 170 – 260 kts		n/a	3
Climb, 20° Attitude, TAS 170 kts		n/a	4
Notes			
1. The complete turn seems slower than the expected and it takes more or less 2 minutes			
2. The complete turn can be operated in 1 minute; 45° bank seems high, yet within normal range			
3. The aircraft response seems realistic; controls seems effective; the overall controllability is accettable			
4. Climb performance are too optimistic and rather unrealistic; Possible errors in the Drag Model			
Further general notes from the pilot:			
a. The aircraft is flyable an it seems quite realistic;			
b. rudder inputs are not very effective; feedback force is fine;			
c. aileron inputs are highly sensitive, it seems that a small displacement of the central stick is already enough to reach full deflection of the ailerons; possible errors in the control interface			

Fig. 20 Test Card of the last flight simulation test performed by the aircraft pilot of ZAV Center of Aviation

VII. Conclusion and Future Developments

In spite of several issues which still affect the model, the project is to be considered successful, since the aim of this work was to build a preliminary issue of a real-time simulation model based on available literature to be integrated and tested on the ReDSim. Nevertheless, the model shall be soon revised in order to improve its fidelity. For instance, the simulation model may well benefit from a full development of the rotors equations of motion (including a dynamic modelling of the rotor inflow), an extensive code debug effort as well as the design of a complete validation procedure to extensively evaluate the model. Consequently the simulation model may be exploited for research activities on handling qualities and design of a stability augmentation control systems.

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