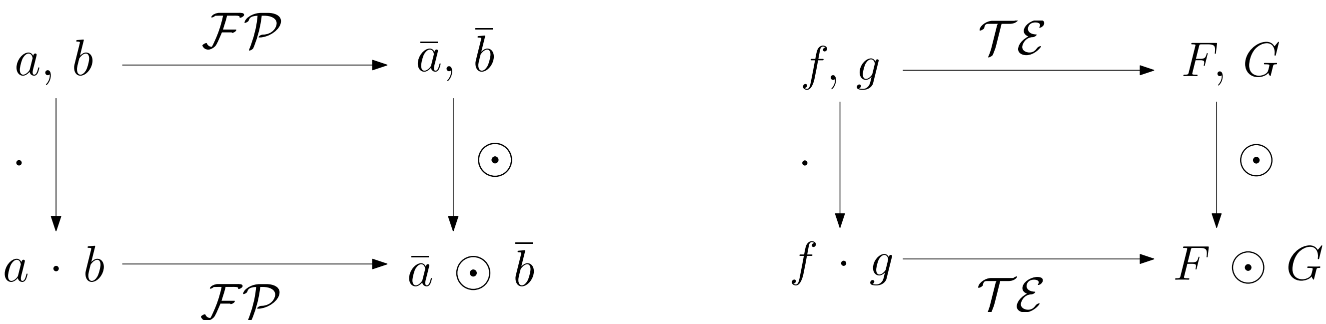
Intern position at ISAE-SUPAERO

Implementation of an Algebraic Library for Orbit Determination

1 The Background

Orbit design, estimation algorithm and orbit determination need time-efficient and performing methods to accurately propagate uncertainties in orbital dynamics. Moreover, many problems in space dynamics, such as sensors models and coordinates transformations, imply the use of non-linear equations whose influence must be taken into account when probability density functions are considered.

In order to quantify and evaluate the evolution of a given initial distribution, several techniques have been developed in the past decades based on the linerization of the non-linear model or on full Monte Carlo simulations. An automatic way to compute derivatives up to a given order is provided by Differential Algebraic (DA) techniques. This method assures the correct evaluation of Taylor expansions in a computer environment by using algebraic predefined operations [1–3].



(a) Commutative diagram for real numbers. $\mathcal{FP}$ is the floating point number conversion operation.

(b) Commutative diagram for functions. $\mathcal{TE}$ is the Taylor expansion conversion operation.

Figure 1: Commutative diagrams for a generic operation $\cdot$ and a generic conversion operation.

This approach is obtained by searching to find the k^{th} Taylor expansion of the flow of the non-linear differential equation in terms of the initial conditions. This polynomial expansion is then used to obtain stochastic information or to perform a Monte Carlo analysis when necessary. The basis of the Taylor Differential Algebra (TDA) approach are found in the use of algebraic means to solve analytical problem. The field pioneer has been Berz [1] with his studies about the Taylor expansion of the flow of ordinary differential equations in terms of the initial conditions to obtain a easy-to-evaluate polynomial surrogate model.

The basic idea of the TDA is to extend the algebraic operation on numbers to function in a computer environment. In a strict sense, neither numbers nor functions can be *exactly* represented in a computer because of the finite amount of information that a machine manages. From a real numbers' point of view, they are approximated by the floating point numbers in a way that the diagram in Figure 1a commutes by firstly applying the operation $\cdot$ on the real number and then converting the result to floating point numbers or by firstly converting to the floating point numbers and then applying the adjoint operation $\odot$. Analogously, a commuting diagram, in Figure 1b, can be defined with a generic function in such a way that the Taylor expansion coefficients extraction and the operation $\cdot$ on function commutes thanks to the definition of an adjoint operation $\odot$. Moreover, by introducing multiplication, scalar multiplication and addiction, the defined mathematical structure turns into a commutative algebra and, by equipping the algebra of a analytic differentiation and integration, the formal definition of a differential algebra is established [1]. The differential algebraic structure is labeled ${}_kD_m$ where k is the Taylor expansion order and m is the number of considered variables.

The first TDA application has been the computation of parametric map in optical systems where particles' trajectories Taylor expansion have been gathered [1]. Recent works have used TDA techniques in space mechanics to solve parametric equations or to take into account uncertainty propagation [2–8].

2 The Internship

In this context the objective of the intern is:

- Understand the previous work on the algebra
- Implement the TDA math library
- Perform validation test and unity test cases

The candidate should have knowledge & knowhow in programming

Date: from October or November 2018
Duration: 5 to 6 months
Remuneration: ∼400€ per month
Contacts: Paolo Panicucci ✉, Emmanuel Zenou ✉

References

[1] Martin Berz. Modern map methods for charged particle optics. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 363(1-2):100–104, 1995.

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[3] Mirco Rasotto, Alessandro Morselli, Alexander Wittig, Mauro Massari, Pierluigi Di Lizia, Roberto Armellin, Celia Valles, and Guillermo Ortega. Differential algebra space toolbox for nonlinear uncertainty propagation in space dynamics. 2016.

[4] Laura Pirovano, Daniele Antonio Santeramo, Alexander Wittig, Roberto Armellin, and Pierluigi Di Lizia. Initial orbit determination based on propagation of orbit sets with Differential Algebra. 2017.

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[6] Roberto Armellin, Pierluigi Di Lizia, Franco Bernelli-Zazzera, and Martin Berz. Asteroid close encounters characterization using differential algebra: the case of Apophis. *Celestial Mechanics and Dynamical Astronomy*, 107(4):451–470, 2010.

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[8] Pierluigi Di Lizia, Mauro Massari, and Francesco Cavenago. Assessment of onboard DA state estimation for spacecraft relative navigation. 2017.