Extension of FDD technique for an automated tracking of modes, using environmental and flight data

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Over the last years, in the field of structural engineering, emphasis has been placed in developing reliable automatic approaches for modal parameter estimations based on Operational Modal Analysis (OMA). The resulting time and cost reduction make these techniques very attractive for the aeronautical field, both for aircraft certification and structural health monitoring purposes. In this paper, a new approach for an automated tracking of modes has been proposed, based on the Frequency Domain Decomposition (FDD) technique and the Modal Assurance Criterion (MAC). Examples of the effectiveness of the methodology are shown, using data from environmental and flight tests on a UAV and from flight tests on a third generation aircraft. The obtained results show a good correlation, if compared to the traditional FDD manual selection procedure, suggesting further improvements in the perspective of real-time tracking of modal parameters.

Nomenclature

OMA Operational Modal Analysis

- *H* Hermitian Matrix
- G Spectral Density Matrix
- Φ Mode Shape Matrix
- Σ Singular Values Matrix
- ϕ Mode Shape
- *PSD* Auto Power Spectral Density
- CSD Cross Power Spectral Density
- SVD Singular Value Decomposition

I. Introduction

The field of structural analysis is constantly evolving to offer increasingly sophisticated and reliable techniques for the estimation of modal parameters. During the last decades of particular importance has been the contribution made by Operational Modal Analysis (OMA), which has allowed to study the dynamic behaviour of structures subjected to real operative conditions. Starting from early 1990s, OMA has drawn great attention in the civil engineering community¹, being applied on buildings like bridges and viaducts, which can be hardly isolated from environmental actions and whose artificial excitation results inappropriate. Furthermore, with the advent of the OMA, testing large systems was no longer a problem. Nowadays such techniques are widely applied in many field of engineering, like in the aeronautical one,² for example when dealing with modal estimates from flight tests. The latter are fundamental whenever changes are made

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to the aircraft, for a structural health monitoring and for the certification of new machines. Available data in literature show how OMA techniques are reliable, but also faster and cheaper with respect to traditional Experimental Modal Analysis (EMA) approaches,¹ because they use algorithms with both rapid implementation and application. However, the process of estimation of modal parameters generally requires an interaction from an experienced user, in particular to distinguish between physical and mathematical modes. In this regard, the actual focus on OMA technique is on trying to reduce costs and times, with a view to automatic or even real-time estimates. For this purpose, some algorithms have been proposed recently, relying primarily on the Frequency Domain Decomposition $(FDD)^3$ and Stochastic Subspace Identification $(SSI)^4$ techniques. However, in some cases, like for the proposed automated method by Brinker⁵ userspecified parameters or threshold values are needed, which may not fit all the cases of study. Otherwise, as for the introduced methodology by E. Reynders⁶, algorithms are often complex and they do not lend themselves well for quick and real-time analyses. Within this paper, the proposed method by C.Ranieri and G. Fabbrocino⁷, which is based on the FDD technique and the Modal Assurance Criterion (MAC), is resumed and new modifications are made in order to obtain a new methodology for an automatic estimation of modal parameters. The algorithm used by C.Ranieri is streamlined and made more effective, with the aim of making a quick automatic tracking of the modes when the structure is working in different operating conditions or in a structural health monitoring perspective. This method is presented as a starting point for future developments and applications for real-time analyses. The details of the technique, and three application cases will be presented below..

II. Automated FDD technique

The proposed automated algorithm, called Frequency Domain Decomposition Tracking Technique (FDDT), is based on the well known FDD technique and it uses the Modal Assurance Criterion for following the evolution of modes, while changing the system operative conditions. In fact, the traditional FDD modal parameter selection procedure can be easily automated if mode shapes, experimental or numerical, are known at an initial reference condition called "Point 1"⁷. In case only experimental data are available, for a certain number of test points, the first dataset is used for a classical FDD analysis, where the modal parameters are manually selected by the operator. By briefly recalling the FDD estimation procedure, the recorded time histories are used for the evaluation of the Power Spectral Density matrix G_{uu} :

$$G_{yy}(\omega) = \begin{vmatrix} PSD_{11}(\omega) & CSD_{12}(\omega) & \dots & CSD_{1M}(\omega) \\ CSD_{21}(\omega) & PSD_{22}(\omega) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ CSD_{M1}(\omega) & \dots & \dots & PSD_{M}M(\omega) \end{vmatrix},$$
(1)

which undergoes a Singular Value Decomposition (SVD) for each frequency line and results expressed as:

$$G_{yy}(\omega)\big|_{\omega=\omega_k} = \Phi_k(\omega_k)\Sigma_k(\omega_k)\Phi_k^H(\omega_k),\tag{2}$$

with $\Sigma_k(\omega_k)$ and $\Phi_k(\omega_k)$ the matrices which contain the singular values and singular vectors, respectively. The first step of the modal parameter estimation lies in the selection of the peaks on the singular value curves, which are graphically represented in function of the frequency values, as shown in Fig.1. This selection is usually performed on the first singular value curve, assuming the hypothesis that near the k-th peak only the k-th mode is dominant. As a consequence the PSD matrix approximates to a matrix with a unitary rank and the first singular vector represents an estimate of the mode shape³. Around the k-th selected peak, a



Figure 1. Example of Singular Value curves and selection of the peaks.

Bell Function is built by evaluating the MAC^a between the singular vector at the k-th natural frequency

^aModal Assurance Criterion. It allows to evaluate the degree of correlation between two modes by performing the following

and the vector of the previous and following frequency lines. While the MAC shows values greater than an imposed threshold (usually not lower than 0.9), the bell follows the behaviour of the singular value curve on which the peak has been selected. The defined Bell Function, which represents the power spectral density function of the k-th Single Degree Of Freedom (SDOF) system, is reverted in Time Domain, using the Discrete Inverse Fourier Transform. In this way, an Autocorrelation Function is obtained, and the damped frequency of the system f_d , associated with the identified mode, can be easily evaluated as it is equal to the inverse of the period of the obtained function. Finally, through the Logarithmic Decrement technique⁸ the i-th modal damping ratio is estimated. The described procedure, which is known as Enhanced Frequency Domain Decomposition (EFDD), is repeated for each selected peak and finally the modal parameters are obtained in term of natural frequencies, damping ratios and mode shapes. Once the modal base at the first reference condition (Point 1) has been estimated, modes can be tracked with the FDDT algorithm for a following test Point 2. The main phases of the approach are schematized in Fig.2 (a). The recorded data at Point 2 are used for the construction of the SVD plot and after that, for each reference modal vector at Point 1, the correspondent natural frequency is considered and the evaluation of the MAC is performed between the selected Point 1 singular vector and all the first singular vectors at Point 2, which belong to frequency lines in the neighbourhood of the reference one (Fig.2 (b)). The frequency line, correspondent to the maximum of the MAC, represents the new natural frequency of the tracked mode.



Figure 2. FDDT automated procedure.

As regarding the range in which the Modal Assurance Criterion is applied a good solution, that has been used within this work, is a choice, for each investigated mode shape, of an interval between -20% and +20% of the natural frequency value at the reference point. However, the percentage of deviation could be changed and optimized for the specific condition and structure under analysis. The algorithm is built in such a way to take into consideration only the frequency values, within the selected interval, at which a peak on the first singular value curve occurs. The reason of this choice lies in the fact that, under the hypotheses of well spaced modes and in an optic of a manual selection of the natural frequencies, the operator generally chooses the peaks only on the first curve of the SVD plot. Once the tracked frequencies have been individuated, the estimation of the structural modal parameters is performed with the traditional EFDD technique. This procedure is then repeated for each mode that has been selected for Point 1. If many test points have been performed, the estimated modal vectors and natural frequencies at the i-th test point are used as reference for the i+1-th configuration. A similar approach has been recently proposed by C.Ranieri $et al.^7$, for an automated tracking of modes of civil buildings, with a structural health monitoring purpose. Such methodology affirms that, for each mode under analysis at a reference condition, a MAC vs frequency plot is evaluated, taking into consideration all the frequency lines of the spectrum at the following test point. The obtained function shows an absolute maximum, which corresponds to the new frequency value. Once

operation:

$$MAC = \frac{|\phi^{(1)^T} * \phi^{(2)}|^2}{(\phi^{(1)^T} * \phi^{(1)}) * (\phi^{(2)^T} * \phi^{(2)})},$$
(3)

selected the frequency lines at the i+1 test point, the algorithm proceeds in evaluating the modal parameters through the EFDD technique. However, this approach presents some limitations. Firstly, the use of the entire spectrum, in the Modal Assurance Criterion, takes much more time than if considering only a frequency interval in the neighbourhood of each reference mode. Furthermore, based on the fact that, in an EFDD approach, the spectral bell functions are built considering all the frequency lines around a selected mode which provide a value of MAC above an imposed threshold, usually greater than 0.9, it is easy to understand that, in the MAC vs frequency plot of a considered mode, the maximum value could not be unique. This condition, which has been detected from a direct application of the method on a set of experimental data, does not allow to clearly detect the shift of the natural frequencies, having at disposal a multiple choice for such values. A last consideration regards the fact that, even in the cases when an absolute maximum value is obtained, such frequency line could not correspond to a peak in the first SVD curve. As a consequence it represents a point that would never be chosen during a manual selection of the peaks. For the application of the proposed methodology in this paper, of great importance is the choice of the reference modal base at the initial condition, which is performed manually by the operator. Furthermore, for a correct tracking close test points must be considered, resulting in small changes in the mode shapes. These aspects will be outlined in the following paragraphs, where three cases of application of the FDDT algorithm will be presented.

III. Results

The proposed methodology has been applied on three case studies in order to validate its efficacy. Firstly, data from environmental tests on a UAV have been used. Afterwards, flight data of the same structure have been considered and finally the FDDT algorithm has been applied on flight data of a third generation aircraft. The results of each case will be presented below.

A. FDDT using environmental data of an UAV

Firstly, the FDDT algorithm has been applied on experimental data relative to a scaled model of the Russian YAK monoplane. The used time histories have been acquired by performing three environmental tests on different configurations of the aircraft. A total number of 23 measurement points has been used, by placing the

accelerometers along the structure, as represented in the geometrical model of Fig.4 (a). With the aim of modifying the natural frequencies of the aircraft, the YAK has been provided with a system able to cause a change in the mass of the structure. Such device consists of elastic balloons, which have been filled with water and inserted into the wing in specific positions. The latter have been connected to tubes that, by regulating the opening of two valves, has allowed to dump the water. The maximum capacity of the each elastic balloon was 300 ml (0.300 Kg), thus allowing to realize three investigated configurations, relative to completely close, half opened and fully opened valves. Such test cases are:

- Case 1: 0 g of added mass for each wing (No Mass Case);
- Case 2: 150 g of added mass for each wing (Half Mass Case);
- Case 3: 300 g of added mass for each wing (Full Mass Case).



Figure 3. Scaled model of YAK 112.

More details about the developed environmental tests can be found in Ref.⁹. For the tracking of the natural frequencies, Configuration 1 has been considered as reference and for it, modes have been manually selected through the traditional procedure of the FDD technique. For the others two configurations both the FDDT and FDD approaches have been used, in order to make a comparison between the obtained results. For all the analyses, time histories of 32768 samples have been used and an 80% overlap with 32 blocks has been applied. With the aim of validating the FDDT approach, attention will be focused only on the natural frequencies and mode shapes of the system, neglecting all the information about modal damping ratios. At the base of this choice, there is the fact that, once individuated the modes, the evaluation of the damping ratios is performed in the same way as for the EFDD technique so any further contribution is provided by the FDDT algorithm.

Five modes have been selected on the first singular value curve of Test Point 1, corresponding to a frequency range equal to [0-24] Hz, focusing the analysis on an interval where a clear dynamic behaviour could be observed. The SVD plots of Fig.5 show the performed manual selection of modes for the three configurations. For the 2-nd and 3-rd cases, only four modes has been taken into consideration, because the peak relative to the first reference frequency has no longer been detected. For the FDDT approach, each reference mode shape vector at the i-th reference point has been compared, through the evaluation of the MAC, with the eigenvectors relative to the peaks on the first singular value curve at the i+1-th configuration, in the proper selected frequency range of analysis. In Tab.1, the estimated natural frequencies from both FDD manual selection



Figure 4. Geometrical model.

and FDDT methods, are reported. As can be easily observed, no changes in the results are detected, according to the used approach. However, the effectiveness of the FDDT technique can be better appreciate by focusing on Tabs.2 and 3. In the first one, for each of the five reference modes of Configuration 1, the MAC values are reported, having considered the eigenvectors of the others modes in the range [0-24] Hz, which correspond to all the peaks on the first singular value curve in this frequency interval. It is easy to notice how the FDDT technique succeeds in tracking the natural frequencies, moving from Configuration 1 to Configuration 2, with the exception of Mode 1. However, this is justified by the fact that for Configuration 2 and 3, the peak relative to this mode does not appear on the first singular value curve any more. For each mode of Tab.2, the difference of the MAC value in correspondence of the tracked frequency (indicated with red



Figure 5. SVD curve plots for the analysed configurations.

Test	1_{ST} Fre	equency[Hz]	2_{ND} Frequency[Hz]		3_{RD} Fr	equency[Hz]	4_{TH} Fre	equency[Hz]	5_{TH} Frequency[Hz]	
\mathbf{Point}	FDD	FDDT	FDD	FDDT	FDD	FDDT	FDD	FDDT	FDD	FDDT
Point1	4.99	4.99	9.97	9.97	12.82	12.82	19.59	18.16	23.50	23.50
Point2	/	/	9.61	9.61	12.82	12.82	18.16	18.16	23.50	23.50
Point3	/	/	9.26	9.26	12.46	12.46	18.16	18.16	23.50	23.50

Table 1. Estimated natural frequencies with both FDD manual selection and FDDT approaches.

	Tr	cacking	1-2		
Reference Mode		Ν	Aodes Point	2	
Point 1	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	/	0.021	0.001	0.001	0.005
2	/	0.984	0.052	0.026	0.024
3	/	0.057	0.987	0.335	0.566
4	/	0.028	0.430	0.892	0.014
5	/	0.017	0.468	0.007	0.995

Table 2.MAC values for Configuration 2.

colour) and in the other peaks (on the same line of the table) is clearly evident, leaving no doubt about the

	Tr	cacking	2-3		
Reference Mode		Ν	Aodes Point	2	
Point 1	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	/	/	/	/	/
2	/	0.385	0.040	0.001	0.011
3	/	0.001	0.843	0.128	0.364
4	/	0.047	0.260	0.350	0.0089
5	/	0.005	0.497	0.003	0.8675

Table 3. MAC values for Configuration 3.

new location of the investigated mode shapes. As a final remark on this first analysis, it can be observed that, the assumed high value of the MAC, when the matching of a mode occurs, indicate a good correlation between the mode shapes at the reference condition and at the following test point, which is the condition on which the FDDT technique is based. By proceeding with the tracking of the natural frequencies, from Configuration 2 to Configuration 3, the reported results in Tab.3 have been obtained. Because Mode 1 has not been detected for Configuration 2, only Modes 2-3-4-5 have been used for this second analysis. In this case, low values in terms of MAC have resulted from the application of the FDDT algorithm, if compared with the previous analysis. In fact, for Modes 2 and 4, a low level of correlation has been detected, switching from the reference condition to the following one, as indicated by the presence of MAC values around 0.4 along the principal diagonal of Tab.3. This result is justified by an evolution of the mode shapes in question, from one configuration to the other. An example of this, is shown in Fig.6, where the mode shapes at 9.97 Hz and 9.26 Hz are reported, respectively for Configuration 1 and 3. From this figure it can be appreciate how, from Configuration 1, the Wing mode shape evolves form a First Symmetrical Bending coupled with a Right Half Wing torsion to a pure First Symmetrical Bending for Configuration 3. This fact suggests that the mass variation between configuration 2 ad 3 has involved consistent changes in the dynamic behaviour of the system. As a consequence, the FDDT algorithm is less effective. However, the deviation between the MAC value at the tracked peak and in the others is still clearly visible from Tab.3.



Figure 6. Mode 2.

B. FDDT using Flight Data of an UAV

The second case study on which the FDDT algorithm has been applied concerns the tracking of modes using flight data of the YAK monoplane. For the tests, four accelerometers have been used and located on the aircraft Vertical Tail, as depicted in Fig.7 (a). Six test points has been analysed, being characterized by different values of the dynamic pressure and angle of attack. Also in this case, the first flight configuration has been analysed with the FDD technique and modes have been manually selected. For the other test points, both FDD and FDDT techniques have been used. For all the analyses, time histories of 16384 samples have been used and a 90% overlap with 16 blocks has been applied. In Fig.7 (b), the obtained SVD curves for the reference Point 1 are shown. Here four modes have been selected and used for the tracking analysis, being relative to peaks at which a clear deformation of the Vertical Tail has occurred. The obtained natural frequencies are reported in Tab.4, while the mode shapes are graphically represented in Fig.9. For the other test points, the FDDT algorithm has been applied for an automatic tracking of modes. For each step, the



Figure 7. FDDT on YAK flight data.

obtained mode shapes at the previous configuration have been used as reference for the following tracking analysis. As already done for the presented case in the previous paragraph, the investigation for each mode has been conducted on a spectrum interval from -20% to +20% of the reference frequency value. The obtained results have been compared with those derived from the application of traditional FDD technique at each test point, in order to appreciate the effectiveness of the proposed method. In Fig. 7 (b), the SVD curves, that have been obtained with the FDD technique are reported, showing the selected modes for the first configuration. Instead, in Fig.8, an example of the frequency lines at which the MAC has been evaluated during an FDDT analysis is provided. The obtained results, from the application of both the FDD and FDDT techniques are reported in Tab.5. For each mode and for each step of the tracking procedure, the value of MAC with which the mode has been detected is reported in order to appreciate the degree of correlation between the modes at the i-th and i+1-th test points. From the obtained results, a good correlation between the FDD and FDDT estimates can be observed. Only for few tracking points, the frequency values are not in agreement for the two analysed approaches. However, even in this cases, the MAC values which correlates the modes at two subsequent flight conditions for the FDDT approach, show an high level of correlation. This fact could be positive or negative. In fact, during the manual selection of the peaks a subjective component is introduced in the estimate, because the operator performs the choice based on the visualization of the mode shapes. In this way, two subsequent peaks, which are characterized by very similar structural deformations could be considered identical and so a wrong choice of the peak could occur. This fact could occur in case of a modal coupling of the element in exam with other structural components. With the evaluation of the MAC, the FDDT algorithm is able to detect even small differences in such modes and to choose the frequency line at which an higher correlation is verified. However, if the reference mode shape has evolved for the subsequent test point and in the neighbourhood of that frequency other coupled modes has maintained the same initial deformation, the FDDT algorithm could find a better correlation with these peaks rather than with the real natural frequency. The solution to avoid these problems could be to increase the number of measurement points and to consider in the modal analysis of complex structures more than one structural element. In this way the modal coupling would be taken into consideration and the uncertainty in the choice of the correct peak would decrease.

Reference	e Modes	⁵ Point	1

Mode	Frequency [Hz]	Mode Shape
1	15.63	1_{ST} Vertical Bending
2	25.88	1_{ST} Torsional
3	30.76	1_{ST} Vertical Bending + 1_{ST} Torsional
4	49.80	2_{ND} Vertical Bending + 1_{ST} Torsional

Table 4. Reference Modes for Test Point 1.



Figure 8. Example of selected frequency lines for the tracking of Mode 4.



Figure 9. Selected Mode Shapes.

IV. FDDT using flight data of a third generation aircraft

Based on the good results provided from the application of the FDDT algorithm on a ultralight aircraft, the effectiveness of the methodology has been validated on a complex structure like a third generation aircraft. For this purpose, data from two cruise flight configurations have been used, recording accelerations in 6 measurement points as shown in Fig.10 (a). For all the analyses, records of 32768 samples, with 16 blocks and an 90% of overlap have been used. Attention will be focused only on the tracking of the natural frequency values because no changes in the estimation of the damping ratio and mode shapes are introduced by the FDDT algorithm with respect to traditional EFDD technique. In the following Tab.6,

Test Point	Mode 1			Mode 2]	Mode 3	3	Mode 4		
	FDD	FDDT	MAC	FDD	FDDT	MAC	FDD	FDDT	MAC	FDD	FDDT	MAC
1	15.63	/	/	25.88	/	/	30.76	/	/	49.80	/	/
2	15.63	15.63	0.98	24.90	24.90	0.99	30.76	30.76	0.99	46.80	46.88	0.96
3	15.63	15.63	0.99	26.86	26.86	0.99	30.76	30.27	0.86	48.34	48.82	0.99
4	15.63	15.14	0.99	27.34	27.34	0.99	30.76	30.76	0.99	48.34	48.34	0.99
5	15.63	15.14	0.99	27.34	27.34	0.99	30.76	31.74	0.95	48.83	48.83	0.99
6	15.63	15.63	0.99	27.34	27.34	0.99	32.23	29.78	0.99	48.83	49.31	0.99

Table 5. Results from the tracking of modes with both FDD and FDDT algorithms.

the obtained tracked frequencies are reported, having considered the first cruise flight configuration as the reference one. For the latter, modes have been manually selected. An example of en estimated mode shape is shown in Fig.10 (b), where a clear dynamic behaviour of the structure can be observed. Based on the



Figure 10. FDDT on a third generation aircraft

results of the tracking procedure, reported in Tab.6, it can be observed that the FDDT algorithm succeeds in tracking the modes in almost all the cases, if compared to the FDD technique. In fact only for three natural frequencies the detected values do not correspond to that one that have been manually selected through the traditional FDD approach. As regarding the first non detected mode, which is the 7-th, a well defined peak was visible on the first SVD curve in case of Test Point 1, whereas for the second flight configuration it has no longer been excited and only a weak peak has been observed at 21.97 Hz. In Fig.11, the tracking of such mode is reported, showing with vertical lines the frequency values at which the MAC has been evaluated for the FDDT algorithm and with a red arrow the frequency at which the mode has moved for the second configuration. Because of the low amplitude of the peak, the FDDT algorithm has not been able to detect the peak. However, during the process of tracking of Mode 7, the peak has been manually inserted by the operator and the final tracked frequency has resulted equal to 21.97 Hz, as the one identified with the FDD technique, showing a MAC value of 0.98. The other modes that have not shown values of the natural frequencies in agreement with those one derived from the manual selection are the 11-th and the 14-th. This fact can be justified considering that, when switching from Test Point 1 to Test Point 2 such modes have presented a slight variation in the mode shape due to a contribution of the external Wing Tank, which was not present in the first configuration. By noticing that all the modes up to 23 Hz are characterised by a 2_{ND} Vertical Wing Bending, like the 11-th and 14-th for the first configuration, the latter could have found a greater correlation with close modes of Test Point 2, which have not presented the induced component of deformation by the Tank. This fact justifies the high values of detected MAC also in case of these modes. Such problem could be arisen with an increment of the number of measured points, on the Wing, but also on the other surfaces of the aircraft. In fact, all the analysed modes are coupled with other components of deformation of the other structural elements of the aircraft. This fact has been clearly observed by performing a Ground Vibration Test on the same structure, before the development of flight tests. All the details about the GVT can be found in Ref.⁸. The extension of the modal analysis to other surfaces could help the process of tracking in following the evolution of the mode shapes. A number of only six measurement points on a large structure like a Wing, is in fact not a good starting point for the application of the method, which is strictly based on the observation of the mode shapes. A final remark on the modal parameter estimate regards those mode shapes that for the second flight configuration have show well defined peaks on the first singular value curve, but have not been detected for Test Point 1. For them, after the process of tracking of the reference modes has ended, the relative peaks have been automatically selected by the FDDT algorithm and confirmed by the operator. In this way all the peaks on the first SVD curve has been considered in the analysis and the final set of detected modes for Point 2 has resulted equal to 19.

Test Point]	Mode	1]	Mode 2	2	Mode 3			Mode 4		
	FDD	FDDT	MAC	FDD	FDDT	MAC	FDD	FDDT	MAC	FDD	FDDT	MAC
1	9.16			12.82			14.28			17.21		
2	9.16	9.16	1.00	12.82	12.82	1.00	15.01	15.01	0.97	16.84	16.84	0.96
Test Point]	Mode	5]	Mode	6		Mode	7]	Mode	8
	FDD	FDDT	MAC	FDD	FDDT	MAC	FDD	FDDT	MAC	FDD	FDDT	MAC
1	19.78			20.87			22.71			24.90		
2	19.04	19.04	0.95	20.87	20.87	0.98	21.97	21.97	0.98	25.27	25.27	0.99
	Mode 9		Mode 10									
Test Point]	Mode	9	N	Aode 1	.0	I	Mode 1	.1	N	Aode 1	2
Test Point] FDD	Mode 9 FDDT	9 MAC	N fdd	Aode 1 FDDT	.0 MAC	N fdd	Mode 1	.1 MAC	N fdd	Aode 1 FDDT	2 MAC
Test Point	FDD 29.30	Mode 9 FDDT	9 MAC	FDD 30.76	Iode 1 FDDT	.0 MAC	FDD 32.23	Mode 1 FDDT	.1 MAC	FDD 35.16	Iode 1 FDDT	2 MAC
Test Point 1 2	FDD 29.30 29.66	Mode 9 FDDT	9 MAC 0.98	FDD 30.76 30.76	Aode 1 FDDT 30.76	.0 MAC 0.98	FDD 32.23 32.96	Aode 1 FDDT 30.76	.1 MAC 0.97	FDD 35.16 34.05	Aode 1 FDDT 34.05	2 MAC 0.99
Test Point 1 2 Test Point	FDD 29.30 29.66	Mode 9 FDDT 29.66 Aode 1	9 MAC 0.98	N FDD 30.76 30.76	Aode 1 FDDT 30.76	00 0.98	FDD 32.23 32.96	Ande 1 FDDT 30.76	.1 MAC 0.97	FDD 35.16 34.05	Aode 1 FDDT 34.05	-2 MAC 0.99
Test Point 1 2 Test Point	FDD 29.30 29.66 N FDD	Mode 9 FDDT 29.66 Aode 1 FDDT	9 MAC 0.98 .3 MAC	N FDD 30.76 30.76 N FDD	Ande 1 FDDT 30.76 Ande 1 FDDT	.0 MAC 0.98 .4 MAC	FDD 32.23 32.96 FDD	Ande 1 FDDT 30.76 Ande 1 FDDT	.1 MAC 0.97 .5 MAC	FDD 35.16 34.05	Ande 1 FDDT 34.05	.2 MAC 0.99
Test Point 1 2 Test Point 1 .	FDD 29.30 29.66 FDD 36.62	Mode 9 FDDT 29.66 Mode 1 FDDT	9 MAC 0.98 .3 MAC	N FDD 30.76 30.76 30.76 N FDD 39.18	Aode 1 FDDT 30.76 Aode 1 FDDT	.0 MAC 0.98 .4 MAC	FDD 32.23 32.96 FDD 41.75	Viode 1 FDDT 30.76 Viode 1 FDDT	.1 MAC 0.97 .5 MAC	FDD 35.16 34.05	fode 1 FDDT 34.05	.2 MAC 0.99

Table 6. Results from the tracking of modes with both FDD and FDDT algorithms.



Figure 11. Tracking of mode at 22.71 Hz.

V. Concluding remarks on FDDT technique

In the previous paragraphs, the effectiveness of the FDDT technique has been proved through the application of the algorithm on three cases of study. The proposed methodology has resulted able to follow the evolution of the natural frequencies, in case of small changes in the correspondent mode shapes, when switching from a configuration to another one. In this regard, the value of the MAC gives an indication on how much each selected mode evolves from a reference configuration to the following one. In the perspective of a manual selection of the peaks on the SVD plot, this technique could suggest the correct choice, resulting in an easier individuation of the natural frequencies. On the contrary, because good results are obtained if close test points are considered and no great changes in the mode shapes occur, the method could be used for real time tracking of the modes. However some limitations still affect this method, suggesting an improvement of the FDDT technique. A first observation regards the choice of the reference modes at Test Point 1, which has to be performed carefully in order to obtain a significant modal base also at the following steps of the analysis. Furthermore, the FDDT algorithm is only able to follow the evolution of the Point 1 reference natural frequencies: if new modes characterize the structural behaviour at the following test points,

they are automatically selected but they have to be approved by the operator in order to be considered for the following tracking points. Another improvement of the technique could regard the way in which the the peaks are considered in the FDDT. In fact, much emphasis could be given to peaks which are closer to the reference frequency and which are better defined or characterized by higher amplitudes. In this regard, the choice of the number of blocks with which the signals are processed, plays a fundamental role in determining the number of detected peaks in a selected frequency range. Finally the algorithm should be made effective also with a limited number of measurement points.

VI. Conclusions

The application of the FDD OMA technique on environmental and flight data has been carried out within this paper. Furthermore, a new approach for an automated tracking of modes has been proposed and applied by using data of a scaled monoplane and of a more complex structure like a third generation aircraft. The Automated Tracking algorithm, named FDDT, has turned out to be effective especially when considering an high number of measurement points, well distributed along the structure and close test points. However, also with few records and closely spaced modes it has provided a good level of correlation, if compared with the manual FDD estimates. Some possible improvements of the technique have been outlined within the paper, with the aim of a future application for a real-time tracking of modes.

Acknowledgments

The author wishes to thank the RSV experimental flight department of the Italian Air Force for making possible the development of tests on the investigated third-generation aircraft.

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