Flexible mode controllers and randomized performance tests for gust load alleviation of a transport aircraft

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Two designs for gust load alleviation of a flexible regional aircraft are presented. The objective of the present work is to encounter two key features of a control system design: (i) actuator limitations in terms of maximum deflection and rate, and (ii) model uncertainties. The design is carried out considering a real and non-linear actuator dynamics. The controllers are tested in both nominal and off-nominal conditions. In the final stage, a probabilistic analysis of the controller performance is presented, assessing how model uncertainties affects the achievable load alleviation.

Nomenclature

H	=	gust gradient
M	=	Mach number
N	=	number of uncertain plant samples generated by a randomized algorithm
p	=	performance probability
p_{est}	=	performance probability estimate
q	=	pitch rate
U_g	=	gust vertical velocity
V	=	airspeed
z	=	flight altitude
z_{cg}	=	center of gravity (CG) vertical translation
α	=	angle of attack
α_g	=	ratio between gust vertical velocity and aircraft airspeed
δ	=	randomized algorithm failure probability
δ_{A_L}	=	left aileron deflection
δ_{A_R}	=	right aileron deflection
δ_E	=	elevator deflection
ε	=	estimate accuracy
θ	=	pitch angle

I. Introduction

IN the last decades, the stringent regulations on pollution emissions and the economic interest of airlines in lowering fuel consumption led to the development of aircraft with lighter and slenderer airframe. An undesired consequence of this configurations is represented by the high deformations that the aircraft experiences during flight, which reduce the lifetime of the aircraft because of the resulting high level of stress. Among the phenomena that may generate these loads into the aircraft structure, a major role is played by gusts, since air is never perfectly steady. Moreover, slender structures have lower vibrational frequencies which might interact with the aircraft rigid-body modes, rendering traditional controllers, commonly designed accounting only for the rigid dynamics, ineffective.

This topic is gaining increasing interest in the aeronautical research field and several Gust Load Alleviation (GLA) systems, based on the most various control architecture, have been developed. The main challenge is that of modelling the flexible dynamics of the aircraft in a reliable way and then to take it into account during the design phase.

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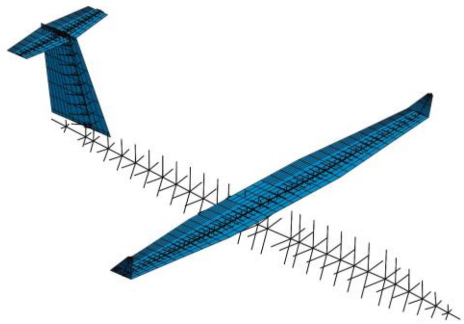

Figure 1. Structural and aerodynamic model of the TP90

Moreover, high performance controllers usually have to face the problem of actuator saturation, which strongly affects and eventually degrades their performance in real environment.

In this work, two approaches to the design of a GLA system for a regional aircraft are presented. Both controllers are based on a $\mathcal{H}_\infty$ controller and take into account the actuator dynamics and saturation. The gust alleviation properties of the two systems are demonstrated for the entire range of gust gradients requested by European aeronautical certifications, at the design point and in a number of off-nominal conditions as well. Finally, a probabilistic analysis of the second control system is carried out, in order to assess its performance variation with respect to uncertainties in the state-space model, thanks to the employment of randomized algorithms for performance verification. An estimate of the performance probability is computed for different performance levels and different uncertainty radii, showing how the controller performance degrades if the system is affected by uncertainties, a very common situation in a real environment.

II. Aircraft and gust modelling

The aircraft considered in this work is a 90-seater regional turboprop concept aircraft with high wing and T-tail, which is being developed within the Clean Sky 2 research program[‡]. An aeroelastic aircraft model is analyzed, in which the interaction between the aerodynamics and structural dynamics are taken into account. The control system is designed for a simplified model, in which only the longitudinal dynamics is included, and verified for a complete system. Moreover, a vertical gust is considered as disturbance and a symmetrical deflection of the ailerons is considered to alleviate gust loads, therefore affecting only the longitudinal dynamics of the aircraft.

A. Flexible aircraft model

The Matlab state-space aeroelastic model of the aircraft was generated starting from a NeoCASS model of the same aircraft. NeoCASS (Next generation Conceptual Aero Structural Sizing)[§] is a free suite of Matlab modules developed by Politecnico di Milano for the aero-structural analysis of a design layout at conceptual design stage. The structural and aerodynamic models are outlined in Fig. 1. A stick model is used to represent the elastic properties of the structure, whereas the aerodynamic mesh is composed by a series of flat panels that enable a Vortex-Lattice Method steady computation or a Doublet-Lattice Method unsteady computation. The model has three independent control surfaces, which are the elevator and the left and right ailerons.

Twelve longitudinal dynamic models and twelve complete models of the flexible aircraft were developed. The three mass configurations considered are Maximum Take-Off Weight (MTOW), Maximum Landing Weight (MLW) and Maximum Zero-Fuel Weight (MZFW). For each of them, four flight conditions are analysed: the cruise design point and the maximum operating speed at Sea Level (SL), corner point and maximum altitude.

The longitudinal aeroelastic model is composed by 66 states:

- Center of Gravity (CG) vertical translation z_{cg},
- pitch angle θ,
- six modal deformations,
- derivatives of the previous states,
- fifty fictitious aerodynamic states created by NeoCASS to simulate the response of the flexible aircraft.

The state vector of the complete models is structured in a similar way with respecto to that of the longitudinal one, including:

- rigid translations of the CG,
- rigid angular positions,
- thirty modal deformation,
- derivatives of the previous states,
- eighty-four aerodynamic fictitious states.

[‡] http://www.cleansky.eu/.

[§] https://www.neocass.org/.

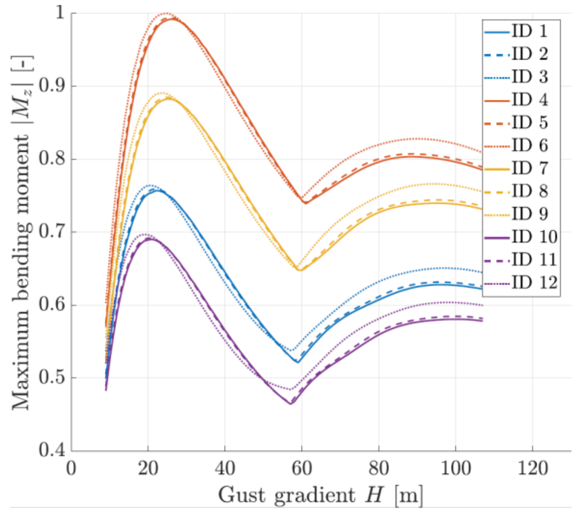

Figure 2. Maximum wing root bending moment for different flight conditions and gust gradients.

Table 1. Maximum wing root bending moment for different flight conditions and gust gradients.

ID	Altitude [m]	***H*** [-]	***V*** [m/s]	Weight	Critical ***H*** [m]
1				MTOW	22
2	6096	0.52	164.6	MLW	22
3				MZFW	20
4				MTOW	26
5	0	0.41	138.9	MLW	26
6				MZFW	25
7				MTOW	25
8	5854	0.58	183.9	MLW	25
9				MZFW	24
10				MTOW	21
11	8534	0.58	177.3	MLW	20
12				MZFW	19

The inputs of the system are the elevator deflection δ_E and the left aileron deflection δ_{A_L}, whereas the right aileron deflection δ_{A_R} is always equal to the latter. A gust input α_g, defined as the ratio between the gust vertical velocity U_g and the aircraft velocity V, is added as disturbance input.

The flexible model has 1200 outputs, which includes displacement, velocities, accelerations and loads of relevant points of the aircraft structure. For the GLA purpose, the wing root loads are those of interest. In particular, as it is usually very difficult to alleviate all of the three wing root moments[1], only the bending moment is considered during the controller design, in accordance with what prescribed by European Aviation Safety Agency (EASA) certification requirements[2].

A numerical model of an Electro-Mechanical Actuator (EMA) actuator was introduced into the aircraft model. The dynamics of the actuator can be described with a second order system. The command saturation is restricted to 15° for the aileron actuator, in order to represent the maximum deflection exploitable for gust alleviation. In accordance with EASA requirements[3], a margin of 15° has to be left free for lateral-directional manoeuvres. A maximum actuation rate of 80 deg/s was specified.

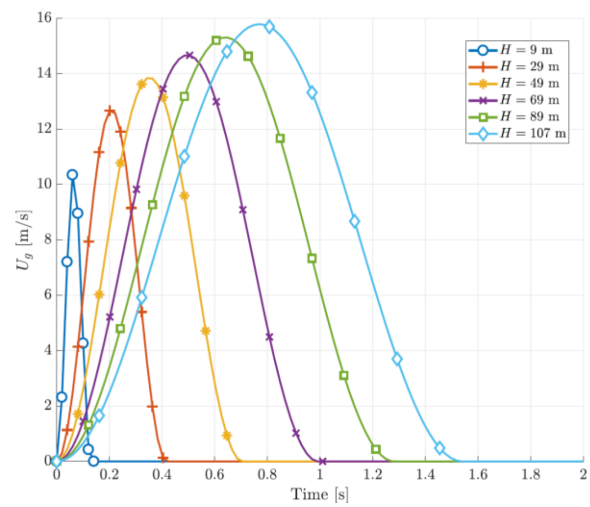

Figure 3. Gust velocity profile for different gust gradients at SL altitude.

B. Gust modelling and critical gust identification

The gust was modelled according to the relevant EASA certification requirements (paragraph CS 25.341[2]), considering a vertical gust load. A key parameter of EASA gust model is the gust gradient H (see Fig. 3) representing the distance, parallel to the aircraft flight path, taken by the gust to reach its peak velocity, which can vary between 9 m and 107 m and influences the gust shape and its peak velocity. The gust peak velocity decreases for increasing flight altitudes, therefore the worst-case is represented by Sea Level (SL) altitude.

The response of the different flexible aircraft models has been analysed with the purpose of identifying the most demanding load condition and the corresponding gust gradient, usually defined as critical gust gradient. The maximum bending moment is found with model number 6, which corresponds to the lowest weight (MZFW), lowest altitude (SL) and lowest Mach number (M = 0.41).

III. LQR controller

Typically, for conventional aircraft, controller designs are based on rigid-aircraft dynamics[1], with the obvious advantage of a simpler controller design due to the lower number of variables to be considered. However it has to be considered that, if the aircraft is sufficiently flexible, the time scales of the first bending/twisting modes and the flight-dynamic response could be comparable, causing the controller to be ineffective as the structural flexibility increases[3].

In order to assess whether the loads generated in the aircraft structure are mainly dependant on the rigid or flexible dynamics, a Linear Quadratic Regulator (LQR) is designed starting from the rigid aircraft model and its GLA properties are then assessed on the flexible one. First a LQR is designed using the rigid aircraft nominal state-space model and tuned according to Bryson's rule[4]. Second, a randomized LQR is defined through the application of randomized algorithms for probabilistic optimal design[5]. This design should overcome the controller performance degradation expected as a consequence of an off-nominal condition application. Indeed, randomized algorithms exploit plant uncertainties during the design process, providing a controller suitable for different flight conditions.

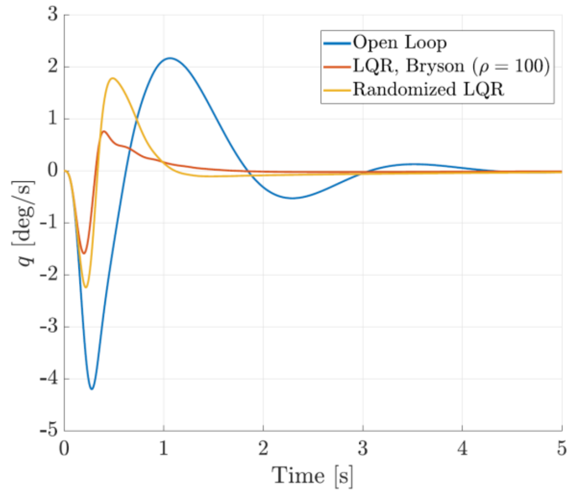

Figure 4. LQR response to a critical gust on the flexible aircraft

Uncertain values in an aircraft state space model are usually due to airspeed, aircraft mass, moments of inertia and aerodynamic derivatives, whereas control derivatives are easier to estimate since they are mainly due to the characteristics of the control system. Airspeed, aircraft mass and moments of inertia are not easy to measure in-flight and their estimates are usually affected by an error which could approximately amount to the 10% of their value. The same holds for the aerodynamic derivatives of the aircraft, which require lots of tests to come to a sufficiently accurate estimate. Moreover, these quantities can significantly vary according to the flight condition. Following these considerations, it was decided to define uncertainties only on the coefficients of the matrix A of the state space model, while B was assumed to be perfectly known, although its coefficients requires the airspeed, aircraft mass and inertia to be computed.

After assessing the stabilization properties of both nominal and randomized LQR on the rigid aircraft model, the two controllers were implemented on the flexible one. The flexible aircraft configuration chosen to test the rigid controller is identified by the ID 6, which corresponds to the flight condition $M = 0.41$ at SL and MZFW, chosen since it is the worst-case scenario in terms of gust load. Three different gust gradients were considered, namely the critical gust gradient (25 m, Fig. 4) and the lower and upper certification boundaries (9 m and 107 m respectively). The analysis shows that the LQR designed on the rigid aircraft dynamics is not suitable for GLA purposes on the flexible aircraft, but it should be suitable for simple manoeuver control, combined with a control system suitably designed for gust load alleviation. The regulator is robust as it is able to control the rigid body motion in off-nominal conditions, nonetheless the flexibility of the aircraft appears to be far from negligible and to be indeed mainly responsible for the internal load generation. The controller acts mainly on the elevator, which is generally used to control the longitudinal rigid dynamics, rather than the ailerons, which, considering motion in the longitudinal plane, usually control the flexible dynamics of the system. It is this latter surface which is indeed mainly used in gust alleviation systems.

IV. $\mathcal{H}_\infty$ controller

Beside the several advantages offered by flexible structures, such as relatively smaller actuators, lower overall mass, faster response and lower energy consumption, an increasing complexity arises from the control system point of view. Among advanced control techniques, the $\mathcal{H}_\infty$ control theory is experiencing a high research effort in the aeronautical field for GLA systems of flexible aircraft[1,6,7] and, even more, in the aerospace fields for satellite attitude control[8,9,10]. The aim of the $\mathcal{H}_\infty$ control theory is the system sensitivity minimization, i.e. the goal is to make a certain performance output z as independent from a disturbance input w as possible. It has to be remarked that the solution to this control problem is optimal with respect to the prescribed cost function (i.e. with respect to the selected performance output) and does not provide the best controller in terms of overall performance or traditional controller performance measures such as settling time, energy expended, etc.

There exist different methods for the solution of the $\mathcal{H}_\infty$ control problem:

- Youla-Kucera parametrization,
- solution of two Riccati equations,

- Linear Matrix Inequalities (LMI) approach.

The first one has the main drawback of often leading to very-high order controllers, whereas the second one requires several simplifying assumptions. The LMI-based approach[11] is basically a reformulation of the Riccati equations and present some additional interesting features with respect to the original Riccati-based approach, mainly resolvability conditions valid for both regular and singular problems and an LMI-based parametrization of all $\mathcal{H}_\infty$-suboptimal controllers, including reduced-order controllers. With this approach, the usual $\mathcal{H}_\infty$ Riccati equations are replaced by Riccati inequalities, which can be expressed as a system of three LMI; the solution set of these inequalities can be used to parametrize all suboptimal $\mathcal{H}_\infty$ controllers, including reduced order ones. Moreover, this method does not require any of the customary regularity assumptions on the rank of D_{zu} and D_{yw} and on j-axis invariant zeros of $P_{zu}(s)$ and $P_{yu}(s)$[12]. The design of a state-feedback $\mathcal{H}_\infty$ control, subject to the performance constraint $\|\mathcal{F}(P,K)\|_\infty < \gamma$ on the transfer function between w and z, is achieved through the resolution of the following set of LMI with Q and Y unknown[15]:

$$\begin{bmatrix} AQ + QA' + B_uY + Y'B_u' & \star & \star \\ C_zQ + D_{zu}Y & -\gamma^2 I & \star \\ B_w' & D_{zw}' & -I \end{bmatrix} \prec 0, \quad Q \prec 0 \tag{1}$$

This consists in an optimization problem whose objective is the minimization of the parameter $\gamma > 0$. Once the LMI are solved, the $\mathcal{H}_\infty$ controller K is given by:

$$K = YQ^{-1} \tag{2}$$

As said before, the objective of this optimization problem is the scalar γ. The software chosen for the solution of the $\mathcal{H}_\infty$ problem, which can be specified within the optional settings of *solvesdp*, is MOSEK[13,14,15], a Matlab-based software package designed for solving large-scale sparse linear problems.

In this work, two $\mathcal{H}_\infty$-based controllers are designed to minimise the bending moment induced by the gust at the wing root. The first one exploits the formulation of a so-called augmented system, which includes the actuator dynamics into the aircraft state-space model, to overcome the actuator saturation issue, whereas the second one features two independent control channels for the aircraft stabilization and the GLA.

A. Augmented system controller

As briefly introduced before, two issues that can be encountered in control system synthesis are represented by model uncertainty and actuator saturation[16]. If these issues are neglected during the design phase, performance degradation or even system instability can be observed when the controller is applied to real systems.

There are mainly two methods for dealing with actuator saturation[17]: an *a-priori* approach which consists in taking control constraints into account in the control design phase and an *a-posteriori* approach which consists in first ignoring actuator saturation, then adding an anti-windup compensator to weaken the adverse influence of saturation. After verifying that an unconstrained $\mathcal{H}_\infty$ controller would exceed the saturation level by almost two orders of magnitude, it was chosen to adopt an *a-priori* approach, including the actuator dynamics into the aircraft state-space model through the formulation af an augmented system.

A state-feedback architecture was chosen for the controller. The non-negligible assumption behind state-feedback design is that the whole state vector is available for direct measurement. In this research, as the state vector includes modal deformations and velocities, some measurements are estimated with a state observer, briefly described in Section IV.

The actuator state space model was built with actuator deflection and rate as outputs. In this way, they can be both incuded in the performance output vector of the augmented system and suitably limited to avoid actuator saturation and reduce the command speed, since commercial aircraft actuators usually features quite low actuation rates. The resulting actuator state-space model is here reported:

$$\begin{cases} \dot{x}_{a,e} = A_{a,e}x_{a,e} + B_{a,e}u_{a,e} \\ \delta_{a,e} = C_{a,e}x_{a,e} \end{cases} \tag{3}$$

where the subscripts a, e indicates either the aileron or the elevator.

According to the methodology presented by Stoica[18], (3) could be included by substitution into the plant standard representation:

$$\begin{cases} \dot{x} = Ax + B_w w + B_u u \\ z = C_z x + D_{zw} w + D_{zu} u \\ y = C_y x + D_{yw} w + D_{yu} u \end{cases} \tag{4}$$

obtaining the following state equation for the augmented system:

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_a \\ \dot{x}_e \end{bmatrix} = \begin{bmatrix} A_p & B_{u,a} C_a & B_{u,e} C_e \\ 0 & A_a & 0 \\ 0 & 0 & A_e \end{bmatrix} \begin{bmatrix} x_p \\ x_a \\ x_e \end{bmatrix} + \begin{bmatrix} 0 \\ B_a \\ 0 \end{bmatrix} u_a + \begin{bmatrix} 0 \\ 0 \\ B_e \end{bmatrix} u_e + \begin{bmatrix} B_w \\ 0 \\ 0 \end{bmatrix} w \tag{5}$$

The performance output equation was modified in the following way to include command deflections and actuation rates in the performance output vector z

$$C_z = \begin{bmatrix} C_{z,p} & 0 \\ 0 & I \end{bmatrix} \tag{6}$$

where $C_{z,p}$ is the matrix which relates the wing root bending moment to the plant state.

The elements of matrix C_z had to be normalized due to the different order of magnitude of the components of z ($M_z \approx 10^6$, $\delta \approx 10^{-1}$), which could cause some of the variables to be "shadowed" by others in the design process of the $\mathcal{H}_\infty$ controller. Similarly to the procedure presented by Cook et al.[1], each row of matrix C_z was divided by the maximum (in absolute sense) value of the elements of that row, ensuring $0 \leq \left|C_{z_{i,j}}\right| \leq 1$. Once scaled, the performance outputs z_i could be weighted to each other by adding a multiplicative coefficient α_i to each row of C_z.

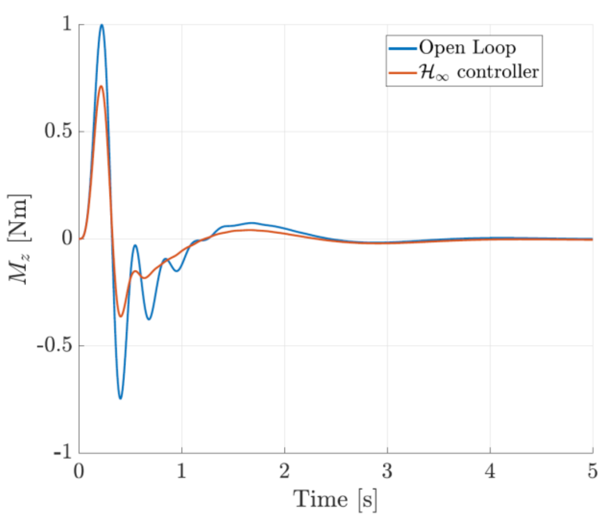

Figure 5. $\mathcal{H}_\infty$ controller response to a critical gust in nominal flight conditions.

The design was carried out considering the worst-case scenario, which corresponds to the flight condition at SL, minimum weight condition (MZFW) and M = 0.41, with a gust gradient H = 25 m. The most critical variable to control was the aileron deflection rate.

The controller achieves a considerable 26.62% load alleviation in critical conditions (Fig. 5), with an aileron deflection lower than 4°. An off-design analysis was then carried out, testing the controller in the whole interval of gust gradient which EASA certification requires to test and in different weight and flight conditions, obtaining the results displayed in Fig. 6. The first thing that stands out is that the controller can not handle large weight variations: indeed, all the aircraft models whose mass configuration corresponded to the MTOW, could not be stabilized by the $\mathcal{H}_\infty$ controller. On the other side, if a smaller variation occur, i.e. the weight is increased from MZFW to MLW, the performance of the controller experiences only minor changes. Variations on the flight conditions are well managed instead, since none of them caused the system to become unstable.

In order to improve the performance of the $\mathcal{H}_\infty$ controller in off-nominal conditions, an alternative design option allowed by the LMI approach was exploited. The logic from which this design procedure originated comes from randomized algorithms. The randomized algorithm for probabilistic optimal design generates a finite number N of

Table 2. Comparison between nominal and randomized $\mathcal{H}_\infty$ controller performance for critical gust.

ID	Weight	M_z alleviation	
		Nominal controller	Randomised controller
1	MTOW	-	-
2	MLW	26.44%	27.32%
3	MZFW	26.84%	27.16%
4	MTOW	-	-
5	MLW	29.03%	29.68%
6	MZFW	28.62%	29.04%
7	MTOW	-	-
8	MLW	30.73%	31.94%
9	MZFW	30.45%	31.38%
10	MTOW	-	-
11	MLW	25.25%	26.31%
12	MZFW	25.79%	26.20%

uncertain plants, defines the corresponding LMI for each sample and then solves the optimization problem subject to the N LMI constraints. This probabilistic design technique resulted to be of difficult application here, mainly due to the definition of realistic uncertainties. Since the flexible aircraft model was already given in state space form, the relation between aircraft characteristics (on which uncertainties should be defined) and the elements of the state space matrices were unknown, making the definition of proper uncertainties very hard, which on turn was fundamental for the formulation of a reasonable optimization. Therefore, it was decided to implement a sort of very simplified randomization of the controller, taking a few samples of the system, represented by the off-nominal conditions state-space models. This allowed to take into account different flight conditions in the controller design phase, introducing reliable uncertainties in the process.

Different sets of flexible aircraft models were defined and investigated, confirming once again that this design is not particularly robust with respect to weight variations. Indeed, including MTOW models in any of the tested sets prevented from obtaining a controller with gust alleviation properties and, in some cases, even led to system instability. Controllers which achieved load alleviation for the MZFW and MLW conditions and did not make the MTOW condition become unstable only reached a very small alleviation. Among all of the configurations analyzed, the best solution included models with the nominal weight (MZFW) but different flight conditions, i.e. models 3, 6, 9 and 12. Conceptually, this corresponds to introducing uncertainties on flight altitude and Mach number with respect to the nominal configuration. The resulting controller was able to alleviate gust loads on the same models as the nominal controller, but with a higher performance in terms of alleviation achieved, as it can be seen in Table 2, where the performance of nominal and randomized controllers in the critical gust case are compared.

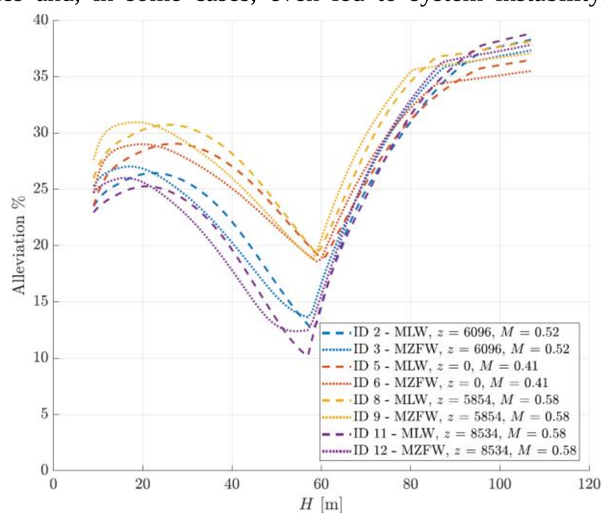

Figure 6. $\mathcal{H}_\infty$ controller response to different gusts in off-nominal flight conditions.

B. Two-channel controller

Passenger aircraft are required to feature optimal flying qualities in order to ensure an adequate level of safety and comfort on board. Gusts and air turbulence, as well as affecting the structure lifetime, induce rigid vibration of the aircraft which should be controlled to prevent an excessive degradation of flying qualities. For this reason, the minimization of the rigid vibration induced by the gust has been assumed as a primary design objective to be achieved in parallel with the alleviation of the structural load. Furthermore, beside this primary objectives, a set of additional requirements was defined with the purpose of overcoming the limits raised during the bibliographical analysis. Simple controllers like state feedback regulators generally provide satisfying performance but only if the model is tested in its design condition. They usually fail when

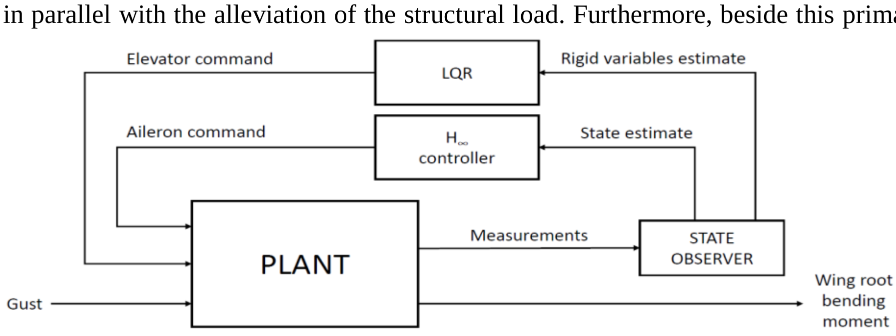

Figure 7. Two-channel controller architecture.

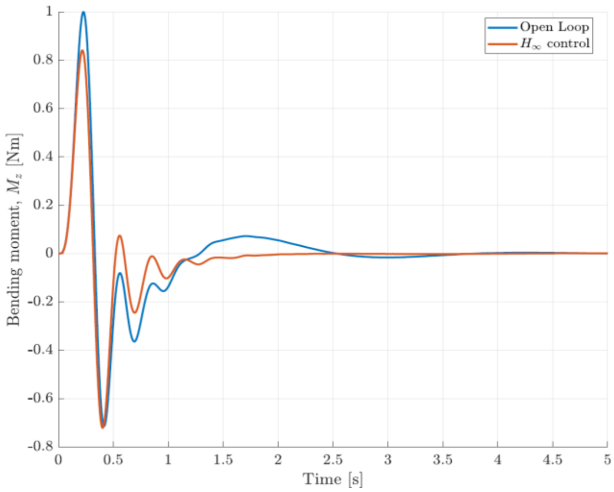

Figure 8. Two channel controller critical gust load alleviation in nominal conditions.

uncertainties are considered or when off-nominal plants are tested. State feedback controllers also require the complete knowledge of the state space which is impossible to obtain in practical applications, unless an heavy instrumentation is installed on the aircraft. Furthermore, actuation systems are frequently considered as ideal components, thus without considering neither their proper dynamics nor the computational delay, resulting in an unrealistic system behavior. Good performance together with accurate actuator models are possible, however considering complicated control laws, for instance adaptive controllers, that do not facilitate the real time computation of the control input. All these aspects, i.e. robustness, architecture simplicity and practical implementability, have been used as drivers throughout the entire design: the controller had to be suitable in most of the aircraft flight conditions, the architecture of the design had to be as simple as possible and the complete actuation dynamics, including computational delay, had to be taken into account. Furthermore, since the system states are not measurable, a set of sensors was defined and a state observer was conceived to overcome the lack of state availability in real time.

The controller architecture consists in two different independent regulators, as shown in Fig. 7. The former is the LQR presented in section III, designed to stabilise the aircraft and regulate the rigid dynamics induced by the gust. The latter is a Static State Feedback (SSF) $\mathcal{H}_\infty$ controller that has been designed through the LMI apporach for alleviating the bending moment at the wing root station of the aircraft. The regulators feature independent actuation systems: the LQR controls only the elevator deflection, whereas the $\mathcal{H}_\infty$ regulator commands the aileron symmetric deflections. The total independence of the controllers derives from the analysis of the LQR performance presented in section III. Indeed, the LQR was able to stabilize the system and ensure a good attenuation of the rigid vibrations exploiting exclusively the elevator deflection, leaving the aileron command available for other purposes. Furthermore, the elevator command employed did not reach in any condition the saturation point, meaning that an additional command could be given to ideally carry out a manoeuvre during the gust. Regarding the synthesis of the $\mathcal{H}_\infty$ controller, the entire set of flight conditions presented in the previous section was exploited for the definition of the LMI. This procedure aimed to achieve the maximum controller robustness and consequently design a SSF controller that was suitable for the entire set of flight conditions.

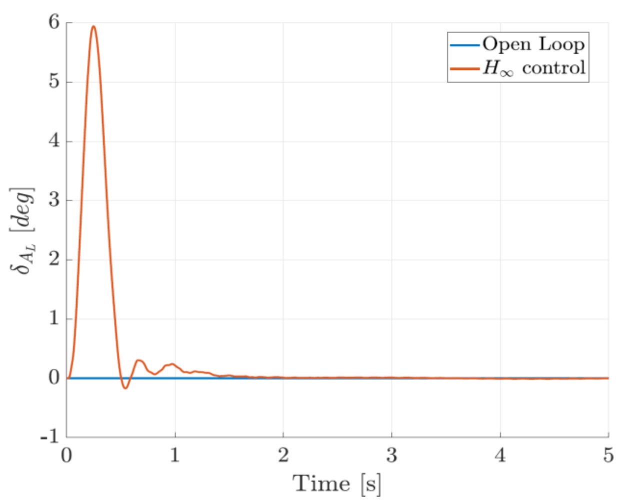

Figure 9. Two channel controller aileron deflection in response to critical gust in nominal conditions.

The architecture includes also a state observer for the estimation of the rigid and flexible states which then are used to compute the independent command actions. The observer is a state partial observer, designed with the methodology proposed by Luenberger[19].

The controller has been tested on both the simplified longitudinal models and the complete-dynamics ones. The first analysis were carried out during the design phase in order to identify the LMI coefficients that provided the best attenuation, whereas the second ones were used to assess the real controller performance. In particular, the controller performance are evaluated in the entire range of gust gradients, testing the aircraft in all the available flight conditions and mass weight. The first analysis aimed to assess the nominal controller performance when the aircraft is perturbed by the critical gust, which corresponds to a gust of gradient H = 25 m when the aircraft is at SL, minimum weight (MZFW) and M = 0.41. The controller provides a 16% alleviation of the first load peak, as it is possible to notice in Fig. 8. Concerning the command inputs, the maximum aileron deflection is equal to 5.77 deg, as shown in Fig. 9.

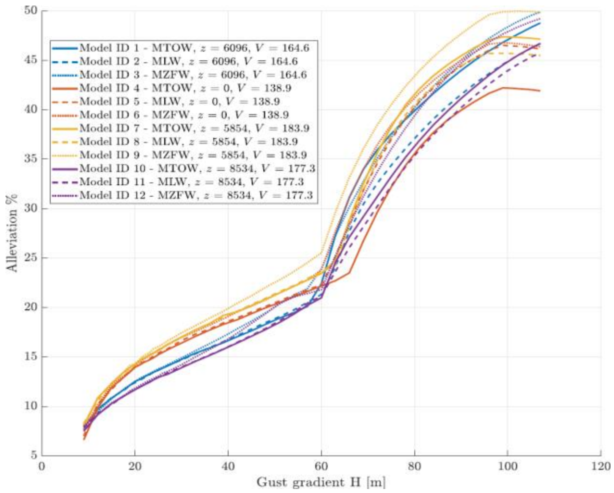

Figure 10. Two channel controller maximum alleviation in off-nominal conditions.

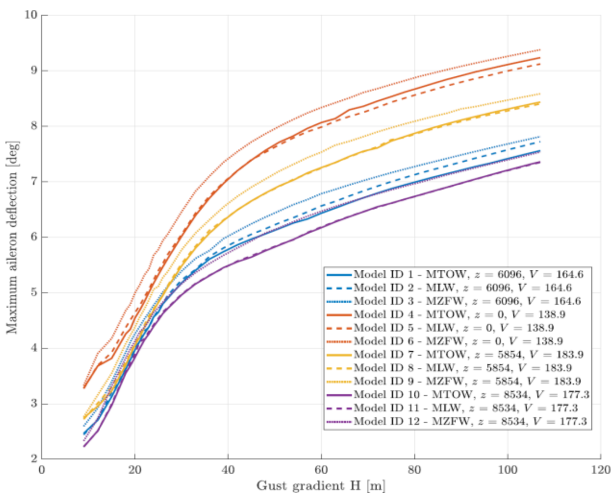

Figure 11. Two channel controller maximum aileron deflection in off-nominal conditions.

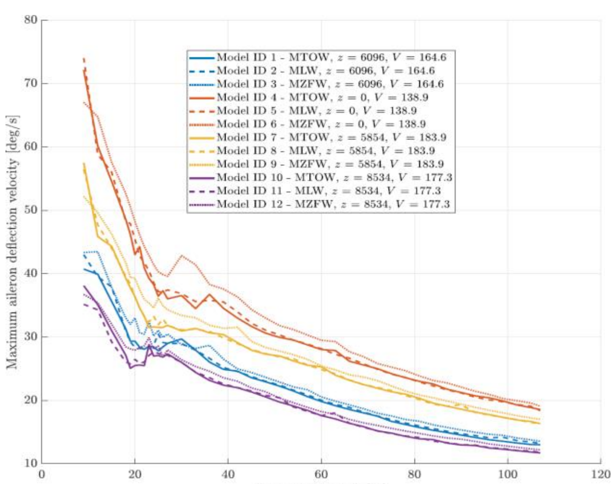

Figure 12. Two channel controller maximum aileron deflection rate in off-nominal conditions.

The controller is then tested on the entire set of flight conditions and the full range of gust gradients. The results are reported in Fig 10 to Fig. 12. The first thing that stands out is that the controller manages to alleviate the maximum load for the totality of the flight conditions and mass configuration, as verifiable in Fig. 10. Starting from a gust of H = 9 m, it can be seen that the attenuation is around the 7% for all of the conditions. Considering critical gust gradients (H included between 20 m and 25 m), the attenuation varies between 13% and 17%, loosing about three percentage points with respect to the simplified models. In conclusion, considering higher gust gradients, for instance H > 50 m, the controller achieves optimal results, reaching alleviations of up to 50%.

The maximum aileron deflection as a function of the gust gradient is compared for the different flight conditions in Fig. 11. Experiencing a maximum deflection of 9.4 deg, the controller never reaches the saturation level for any of the gust gradient, showing compliance with EASA requirements, which set the maximum allowed aileron deflection to 15 deg. For performance assessment, it is worthy to verify that the aileron deflection rate (Fig. 12) does not reach the actuator saturation speed of 80 deg/s for any of the flight conditions.

V. Probabilistic performance verification

The controller robustness with respect to model uncertainties was verified at last, aiming to characterize the controller performance in presence of uncertainties in the state-space model, representative of the real system behaviour, in which several parameters affecting the controller response are not perfectly measurable, such as mass condition, centre of gravity position, real aerodynamic coefficients, etc.

The robustness assessment is carried out by applying the randomized algorithms for performance verification, a method that allows to assess the robustness in a probabilistic sense with a finite number N_{sim} of simulations. In particular, the procedure returns, with a probability $1-\delta$, an estimate p_{est} of the performance probability p with accuracy ε. Applying this procedure to a GLA problem, the performance level can be defined as a minimum load alleviation and therefore what the procedure will compute is an estimate of the probability of the achievement of a desired load alleviation with probability $1-\delta$ and with accuracy ε.

In this work, the Matlab toolbox RACT is used to implement randomized algorithms for probabilistic performance verification. In particular, the following steps are followed:

1. compute the number N of samples required to satisfy the probability constraint $|p - p_{est}| \leq \varepsilon$, given by the Chernoff bound:

$$N = \frac{1}{2\varepsilon^2} \log \frac{2}{\delta} \tag{7}$$

2. draw N independent samples $\Delta^{(1)}, \dots, \Delta^{(N)}$ of the state-space model;
3. return the empirical probability p_{est}:

$$p_{est} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{I}\left[J\left(\Delta^{(i)}\right) \leq \gamma \right] \tag{8}$$

where $\mathbb{I}[\cdot] = 1$ when the argument is true, otherwise it is equal to zero.

The probabilistic assessment of the controller performance aimed at characterising how the alleviation achieved decreases with increasing uncertainty radius. Therefore, the first thing to be established was the uncertainty radius range to be investigated. This was extended from 1% of the nominal value of each perturbed matrix coefficient to the 15%, as suggested by Polyak and Tempo[20]. Since the analytical expressions of the elements of the state-space models were not available and affecting the matrix element by element was not possible, the uncertainty range has been intended as the maximum possible percentage variation allowed to each element of the nominal matrix. Indeed, a matrix of aleatory values $A_{unc_{i,j}}$ included in the desired range was constructed and used to compute the sample as follows:

$$A_{unc_{i,j}} = A_{nom(i,j)} + \alpha_{unc}\, A_{nom(i,j)} \tag{9}$$

However, this process perturbs the entire state matrix, including the rows that ensure the right dynamic relations between states (the second half of the state vector consists in the derivatives of the first half space). Indeed, the process was corrected by defining a matrix D_{unc}, whose elements are zeros in correspondence of each rows that define a simple derivation between two states. In conclusion, the $A_{unc_{i,j}}$ element of the uncertain model matrix is defined as:

$$A_{unc(i,j)} = A_{nom(i,j)} + D_{unc(i,j)}\, \alpha_{unc}\, A_{nom(i,j)} \tag{10}$$

Once created the N_{sim} samples with these procedure, it was possible to carry out the simulations and estimate the probability of success and failure as functions of the uncertainty range. The probability of positive alleviation lingers above the 99% until an uncertainty range of 10%, dropping to a value of 94% for an uncertainty range of 15%.

Figure 13 shows the level of alleviation as a function of the uncertain radius: a minimum alleviation of 13% is guaranteed, even if the performance of degradation due to unceratintiess is not changing step-by-step. Moreover, it can be demonstrated that the probability of stabilising the system without load alleviation is equal to zero in the entire range of uncertainty radius.

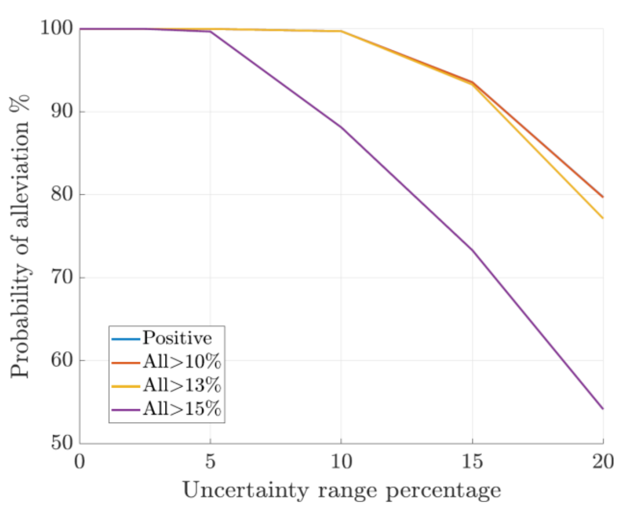

Figure 13. Two channel controller maximum aileron deflection rate in off-nominal conditions.

VI. Conclusion

Two different control systems for GLA of a flexible aircraft were presented. The first one, based on a $\mathcal{H}_\infty$ controller, exploits the formulation of an augmented state-space system, which included the actuator dynamics, to solve the common issue of actuator saturation. The controller featured very interesting performance at the design point and in some off-nominal conditions. However, the controller was not able to alleviate gust loads in all of the tested conditions, leading the system to instability in the MTOW flight conditions.

A second design is then proposed, featuring an LQR for the elevator control and aircraft stabilization in parallel to a $\mathcal{H}_\infty$ controller which provides symmetric input to the ailerons for gust alleviation. This second controller, designed on an aircraft longitudinal state-space model and validated on a full model, achieves a considerable performance in terms of load alleviation for the entire rane of gust gradients required by the certification and for all the twelve flight conditions tested. Thanks to the application of randomized algorithms, the controller performance in presence of system uncertainties was assessed, investigating how the probability of a target alleviation decreases with increasing uncertainties. This analysis showed an exceptional robustness for model uncertainties of up to 10%. It should be noted that these results, comparable with those of adaptive controllers, were obtained with a much simpler architecture, i.e. a state-feedback one, which does not require any particular computational capability.

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References

[1]Cook, R. G., Palacios, R., and Goulart, P., "Robust Gust Alleviation and Stabilization of Very Flexible Aircraft", *AIAA Journal*, Vol. 51, No. 2, Feb. 2013, pp. 330-340.

[2]"Certification Specifications and Acceptable Means of Compliance for Large Aeroplanes CS-25", Amendment 16, European Aviation Safety Agency, Brussels, 12 Mar. 2015.

[3]Yamashiro, K., and Stirling, R., "Reduction of Flight Control System/Structural Model Interaction", *AIAA Atmospheric Flight Mechanics Conference and Exhibit*, AIAA, 2007.

[4]España, J. P., "Lecture Notes on LQR/LQG Controller Design", *Knowledge Creation Diffusion Utilization*, 2005.

[5]Motwani, R., Raghavan, P., *Randomized algorithms*, Cambridge University Press, New York, NY, 1995.

[6]Aouf, N., Boulet, B., and Botez, R., "$\mathcal{H}_2$ and $\mathcal{H}_\infty$-Optimal Gust Load Alleviation for a Flexible Aircraft", *Proceedings of the American Control Conference*, 2008.

[7]Jannson, N., and Eller, D., "Robust Turbulence Load Alleviation", *IFASD 2011: International Forum on Aeroelasticity and Structural Dynamics*, 2011

[8]Chamanbaz, M., Dabbene, F., Peaucelle, D., Pittet, C., and Tempo, R., "Randomized and Robust Methods for Uncertain Systems Using R-ROMULOC,With Applications to DEMETER Satellite Benchmark", *Aerospace Lab Journal*, 2016.

[9]Méndez Cubillos, X. C., and Gadelha de Souza, L. C., "Using of H-Infinity Control Method in Attitude Control System of Rigid-Flexible Satellite", *Mathematical Problems in Engineering*, Vol. 2009, 2009, pp. 1-9.

[10]Bai, H., Zhou, Y., Sun, H., and Zeng, J., "Observer-Based Nonlinear H-Infinity Attitude Control for a Flexible Satellite", *IET Control Theory & Applications*, 2017.

[11]Gahinet, P. M., and Apkarian, P., "A linear matrix inequality approach to $\mathcal{H}_\infty$ control", *International Journal of Robust and Nonlinear Control*, Vol. 4, 1994, pp. 421-448.

[12]Doyle, J., Glover, K., Khargonekar, P., and Francis B., "State-space solutions to standard $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems", *IEEE Transactions on Automatic Control*, Vol. 8, No. 8, 1989, pp. 831-847.

[13]Andersen, E. D., and Ye, Y., "A computational study of the homogeneous algorithm for large-scale convex optimization", *Computational Optimization and Applications*, Vol. 10, 1998, pp. 243-269.

[14]Andersen, E. D., and Andersen, K. D., "The Mosek interior point optimizer for linear programming: an implementation of the homogeneous algorithm", in Frenk, H., Roos, K., Terlaky, T., and Zhang, S., *High performance Optimization*, Kluwer Academic Publisher, 2000, pp. 197-232.

[15]Andersen, E. D., Roos, C., and Terlaky, T., "On implementing a primal-dual interior-point method for conic quadratic optimization", *Math. Programming*, Vol. 95, 2003, pp. 249-277.

[16]Cao, Y. Y., Lin, Z., and Ward, D. G., "H_∞ Anti-Windup Design for Linear Systems Subject to Input Saturation", *AIAA Guidance, Navigation, and Control Conference and Exhibit*, 2001.

[17]Turner, M. C., Herrmann, G., and Postlethwaite, I., "Incorporating Robustness Requirements Into Antiwindup Design", *IEEE Transactions on Automatic Control*, Vol. 52, No. 10, IEEE, 2007, pp. 1842–1855.

[18]Stoica, A., "Optimal H-Infinity Design for a Control System for the Angle of Attack", *1997 European Control Conference*, 1997, pp. 337-342.

[19]Luenberger, D. G., *Introduction to dynamic systems - Theory, models and applications*, John Wiley and Sons, 1979.

[20]Polyak, B. T., and Tempo, R., "Probabilistic Robust Design with Linear Quadratic Regulators", *System & Control Letters*, Vol. 43, No. 5, 2001, pp. 343-353.